Supplementary Material for Adaptive Protection for Lossy Representation

Xiaofei He, *Senior Member, IEEE*, Chuyuan Zhang, Liuyun Zhang, *Member, IEEE*, and Xuanwen Huang, *Member, IEEE*

By requiring the gradient of ϕ with respect to B^T vanish, we have

$$\begin{aligned} &\frac{\partial \phi}{\partial B^T} = 0 \\ \Rightarrow\ & B^T \widetilde{X}^T A A^T \widetilde{X} + B^T - A^T \widetilde{X} = 0 \\ \Rightarrow\ & B = (\widetilde{X}^T A A^T \widetilde{X} + \lambda I)^{-1} \widetilde{X}^T A. \end{aligned} \tag{2}$$

Substituting Eq. (2) into Eq. (1) and noticing that $\mathrm{Tr}(AB) = \mathrm{Tr}(BA)$, we have

$$\begin{aligned} &\mathrm{Tr}(A^T \widetilde{X} B) \\ =\ & \mathrm{Tr}\Big(A^T \widetilde{X} \big(\widetilde{X}^T A A^T \widetilde{X} + \lambda I\big)^{-1} \widetilde{X}^T A\Big) \\ =\ & \mathrm{Tr}\Big(\big(\widetilde{X}^T A A^T \widetilde{X} + \lambda I\big)^{-1} \widetilde{X}^T A A^T \widetilde{X}\Big) \\ =\ & \mathrm{Tr}\Big(\big(\widetilde{X}^T A A^T \widetilde{X} + \lambda I\big)^{-1} \big(\widetilde{X}^T A A^T \widetilde{X} + \lambda I - \lambda I\big)\Big) \\ =\ & n - \lambda \mathrm{Tr}\Big(\big(\widetilde{X}^T A A^T \widetilde{X} + \lambda I\big)^{-1}\Big), \end{aligned} \tag{3}$$

and,

$$\begin{aligned} &\mathrm{Tr}(A^T \widetilde{X} B B^T \widetilde{X}^T A) + \lambda \mathrm{Tr}(B B^T) \\ =\ & \mathrm{Tr}(B B^T \widetilde{X}^T A A^T \widetilde{X}) + \lambda \mathrm{Tr}(B B^T) \\ =\ & \mathrm{Tr}\Big(B B^T \big(\widetilde{X}^T A A^T \widetilde{X} + \lambda I\big)\Big) \\ =\ & \mathrm{Tr}\Big(\big(\widetilde{X}^T A A^T \widetilde{X} + \lambda I\big)^{-1} \widetilde{X}^T A A^T \widetilde{X} \big(\widetilde{X}^T A A^T \widetilde{X} + \lambda I\big)^{-1} \\ & \qquad \big(\widetilde{X}^T A A^T \widetilde{X} + \lambda I\big)\Big) \\ =\ & \mathrm{Tr}\Big(\big(\widetilde{X}^T A A^T \widetilde{X} + \lambda I\big)^{-1} \widetilde{X}^T A A^T \widetilde{X}\Big) \\ =\ & n - \lambda \mathrm{Tr}\Big(\big(\widetilde{X}^T A A^T \widetilde{X} + \lambda I\big)^{-1}\Big). \end{aligned} \tag{4}$$

Finally, we have

$$\phi = k - n + \lambda \mathrm{Tr}\Big(\big(\widetilde{X}^T A A^T \widetilde{X} + \lambda I\big)^{-1}\Big). \tag{5}$$

Thus, the optimal A is given by solving the following problem:

$$\min_A \mathrm{Tr}\Big(\big(\widetilde{X}^T A A^T \widetilde{X} + \lambda I\big)^{-1}\Big). \tag{6}$$

This completes the proof.

References

[1] S. Boyd and L. Vandenberghe. *Convex Opti ization*. Cambridge University Press, 2004.