

Supplemental Material for Abstracts or Oral Presentations

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By requiring the gradient of ϕ with respect to B^T vanish, we have

$$\begin{aligned} \frac{\partial \phi}{\partial B^T} &= 0 \\ \Rightarrow B^T \tilde{X}^T A A^T \tilde{X} + B^T - A^T \tilde{X} &= 0 \\ \Rightarrow B &= (\tilde{X}^T A A^T \tilde{X} + \lambda I)^{-1} \tilde{X}^T A. \end{aligned} \quad (2)$$

Substituting Eq. (2) into Eq. (1) and noticing that $\text{Tr}(AB) = \text{Tr}(BA)$, we have

$$\begin{aligned} &\text{Tr}(A^T \tilde{X} B) \\ &= \text{Tr}\left(A^T \tilde{X} (\tilde{X}^T A A^T \tilde{X} + \lambda I)^{-1} \tilde{X}^T A\right) \\ &= \text{Tr}\left((\tilde{X}^T A A^T \tilde{X} + \lambda I)^{-1} \tilde{X}^T A A^T \tilde{X}\right) \\ &= \text{Tr}\left((\tilde{X}^T A A^T \tilde{X} + \lambda I)^{-1} (\tilde{X}^T A A^T \tilde{X} + \lambda I - \lambda I)\right) \\ &= n - \lambda \text{Tr}\left((\tilde{X}^T A A^T \tilde{X} + \lambda I)^{-1}\right), \end{aligned} \quad (3)$$

and,

$$\begin{aligned} &\text{Tr}(A^T \tilde{X} B B^T \tilde{X}^T A) + \lambda \text{Tr}(B B^T) \\ &= \text{Tr}(B B^T \tilde{X}^T A A^T \tilde{X}) + \lambda \text{Tr}(B B^T) \\ &= \text{Tr}\left(B B^T (\tilde{X}^T A A^T \tilde{X} + \lambda I)\right) \\ &= \text{Tr}\left((\tilde{X}^T A A^T \tilde{X} + \lambda I)^{-1} \tilde{X}^T A A^T \tilde{X} (\tilde{X}^T A A^T \tilde{X} + \lambda I)^{-1} \right. \\ &\quad \left. (\tilde{X}^T A A^T \tilde{X} + \lambda I)\right) \\ &= \text{Tr}\left((\tilde{X}^T A A^T \tilde{X} + \lambda I)^{-1} \tilde{X}^T A A^T \tilde{X}\right) \\ &= n - \lambda \text{Tr}\left((\tilde{X}^T A A^T \tilde{X} + \lambda I)^{-1}\right). \end{aligned} \quad (4)$$

Finally, we have

$$\phi = k - n + \lambda \text{Tr}\left((\tilde{X}^T A A^T \tilde{X} + \lambda I)^{-1}\right). \quad (5)$$

Thus, the optimal A is given by solving the following problem:

$$\min_A \text{Tr}\left((\tilde{X}^T A A^T \tilde{X} + \lambda I)^{-1}\right). \quad (6)$$

This completes the proof.

REFERENCES

- [1] S. Boyd and L. Vandenberghe. *Convex Optimization*. Cambridge University Press, 2004.