

# Constrained Laplacian Eigenmap for dimensionality reduction

Chun Chen, Lijun Zhang, Jiajun Bu, Can Wang<sup>\*</sup>, Wei Chen

Zhejiang Key Laboratory of Service Robot, College of Computer Science, Zhejiang University, Hangzhou 310027, China

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## ABSTRACT

Dimensionality reduction is a commonly used tool in machine learning, especially when dealing with high dimensional data. We consider semi-supervised graph based dimensionality reduction in this paper, and a novel dimensionality reduction algorithm called constrained Laplacian Eigenmap (CLE) is proposed. Suppose the data set contains  $r$  classes, and for each class we have some labeled points. CLE maps each data point into  $r$  different lines, and each map  $i$  tries to separate points belonging to class  $i$  from others by using label information. CLE constrains the solution space of Laplacian Eigenmap only to contain embedding results that are consistent with the labels. Then, each point is represented as a  $r$ -dimensional vector. Labeled points belonging to the same class are merged together, labeled points belonging to different classes are separated, and similar points are close to one another. We perform semi-supervised document clustering using CLE on two standard corpora. Experimental results show that CLE is very effective.

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## 1. Introduction

In many real life applications, one is often confronted with high-dimensional data (e.g. documents, images). The infamous *curse of dimensionality* states that the performance of most machine learning algorithms degrades rapidly both in effectivity and efficiency as the dimensionality increases [15]. Dimensionality reduction allows one to represent the data in a lower dimensional space, and more importantly, reveal the intrinsic structure of the data [8,45,44,34]. The most popular techniques include principal component analysis (PCA), non-negative matrix factorization (NMF), and graph based dimensionality reduction.

Principal component analysis (PCA) [5,31,30] reduces the dimensionality of the data by finding a few orthogonal linear projections such that the variance of the projected data is maximized. In fact, it turns out that these projections are just the leading eigenvectors of the data's covariance matrix, which are called principal components. PCA is globally optimal in the sense that those projections best preserve the global Euclidean structure of the data. However, it can only discover linear

be handled perfectly. For multi-class problems, the algorithm does map points from the same class into the same point, but cannot guarantee that data points from different classes are mapped separately.

In this paper, we proposed a novel dimensionality reduction algorithm called constrained Laplacian Eigenmap (CLE), which supports multi-class problems. CLE aims to find the projection which respects the intrinsic geometrical structure inferred from all the data points, and also consists with labels. Suppose the data set contains  $r$  classes, and for each class we have some labeled points. CLE maps each data point into  $r$  different lines, and each map  $i$  tries to separate points belonging to class  $i$  from others by using label information. The mapping is performed by constraining the solution space of Laplacian Eigenmap to consist with labels. In the resulting  $r$ -dimensional space, labeled points belonging to the same class are merged together, labeled points belonging to different classes are separated, and similar points are close to one another.

We perform document clustering using CLE on two standard corpora: TDT2 and Reuters-21578. Experimental results show that this clustering model is very effective, and CLE becomes more competitive as the percentage of labeled documents or the number of clusters increases.

The paper is organized as follows: in Section 2, we give a brief review of related works. Our constrained Laplacian Eigenmap (CLE) algorithm is introduced in Section 3. The experimental results are presented in Section 4. Finally, we provide some concluding remarks and suggestions for future work in Section 5.

## 2. Related works

### 2.1. Principal component analysis (PCA)

PCA [5,31,30] can be defined in terms of the orthogonal projections which maximize the variance in the projected space. Given a set of  $n$ -dimensional data points  $\mathbf{x}_1, \dots, \mathbf{x}_m$ , we first consider the projection onto a one-dimensional space using a  $n$ -dimensional vector  $\mathbf{u}$ . Then, we have the following optimization problem:

$$\mathbf{u}_{opt} = \operatorname{argmax}_{\mathbf{u}} \mathbf{u}^T S \mathbf{u} \quad (1)$$

with the constraint

$$\mathbf{u}^T \mathbf{u} = 1$$

where  $S$  is the data covariance matrix defined by

$$S = \frac{1}{m} \sum_{i=1}^m (\mathbf{x}_i - \bar{\mathbf{x}})(\mathbf{x}_i - \bar{\mathbf{x}})^T \quad (2)$$

It can be proved that  $\mathbf{u}_{opt}$  equals to the eigenvector of  $S$  that having the largest eigenvalue  $\lambda_1$ . This eigenvector is called as the first principal component. If we consider the general case of a  $r$ -dimensional projection space, the optimal linear projections for which the variance of the projected data is maximized are just the  $r$  leading eigenvectors  $\mathbf{u}_1, \dots, \mathbf{u}_r$  of  $S$ .

PCA is closely related to latent semantic indexing (LSI) [14] which projects documents onto a lower dimensional space through singular value decomposition (SVD).

### 2.2. Non-negative matrix factorization (NMF)

NMF [22,40,23] aims to find the non-negative factorization of

### 2.5. Constrained spectral clustering (CSC)

CSC is one closely related topic, since most clustering algorithms perform dimensionality reduction before clustering. We discuss two typical algorithms as follows.

Coleman et al. [13] have proposed one constrained spectral clustering algorithm, which supports must-link and cannot-link advices. Different from pervious methods, the proposed algorithm can handle inconsistent advices. However, their algorithm can only be applied to two-class clustering problems. Ji et al. [20] have proposed a document clustering model that enables the user to provide must-link constraints. Specifically, they introduce a constraint matrix  $U$  to encode user's prior knowledge. A penalty term that depends on  $U^T U$  is added to the objective function of spectral clustering. It is easy to check that  $U^T U$  is just the *graph Laplacian* of one graph constructed by adding an edge between any two must-link points. So, this algorithm is essentially equivalent to the semi-supervised graph embedding algorithm that increases the weight of the edge between any two must-link points.

### 3. Constrained Laplacian Eigenmap

**Definition 1.** The semi-supervised dimensionality reduction problem considered in this paper is defined as:

1. *Input:* a set of  $n$ -dimensional data points  $\mathbf{x}_1, \dots, \mathbf{x}_m$  that belong to  $r$  classes, and for each class  $i$ , there are  $m_i$  points labeled.
2. *Output:* the projection that respects the intrinsic geometrical structure inferred from all the data points and also consists with labels.

Denote class  $i$  as  $c_i$ , and the set of labeled points belonging to  $c_i$  as  $l_i$ .

#### 3.1. Motivation

We can translate one multi-class dimensionality reduction problem into a set of two-class problems. This idea can be justified by considering the indicator matrix.  $M \in \{0, 1\}^{m \times r}$  is an indicator matrix if  $M_{ij} = 1$  iff  $\mathbf{x}_i \in c_j$ . Checking columns of  $M$  individually, we can see that the  $i$ -th column of  $M$  can only separate points belonging to  $c_i$  from others, so it can be seen as the embedding result of a two-class dimensionality reduction problem. However, by combining  $r$  such vectors to form  $M$ , it is sufficient to solve  $r$ -class problems, no matter  $r$  is 2 or greater.

Based on the above discussion, we propose a novel semi-supervised dimensionality reduction algorithm called constrained Laplacian Eigenmap (CLE). If the data set contains  $r$  classes, CLE maps each data point into  $r$  different lines, and each map  $i$  tries to separate points belonging to  $c_i$  from others by using the label information. The mapping is performed based on Laplacian Eigenmap, whose solution space is constrained by *modifying the similarity matrix* and *using the subspace trick*. Putting the result of each map as a column, we can form a matrix  $Y \in \mathbf{R}^{m \times r}$ , where the  $i$ -th row gives the embedding coordinate of the  $i$ -th point.

#### 3.2. Constraining the solution space of LE

Suppose we are going to map each point into the  $k$ -th line. The data set is treated as containing two classes:  $c_k$  and  $\bar{c}_k$ . Then, this map is required to separate points belonging to  $c_k$  from others by using the label information. We use two steps to constrain the solution space of Laplacian Eigenmap. Firstly, we incorporate label

information into the graph structure by modifying the similarity matrix. Since we are facing a two-class dimensionality reduction problem, the subspace trick is used later.

##### 3.2.1. Modifying the similarity matrix

Since the data set is treated as containing two classes, we modify the similarity matrix  $W$  to make it more consist with this assumption. Assume the maximum similarity is 1.  $W$  is modified as follows:

1.  $\forall i, j$ , if  $\mathbf{x}_i, \mathbf{x}_j \in l_k$ , set  $W_{ij} = 1$ .
2.  $\forall i, j$ , if  $\mathbf{x}_i \in l_a, \mathbf{x}_j \in l_b, a \neq k$ , and  $b \neq k$ , set  $W_{ij} = 1$ .
3.  $\forall i, j$ , if  $\mathbf{x}_i \in l_k, \mathbf{x}_j \in l_a$ , and  $a \neq k$ , set  $W_{ij} = W_{ji} = 0$ .

After modification, within class ( $c_k$  or  $\bar{c}_k$ ) links become tighter, and between class ( $c_k$  and  $\bar{c}_k$ ) links become looser. Unlike previous algorithm [21], the modification here is different for different map. Let  $W_k$  be the similarity matrix after modification,  $D_k$  be the corresponding diagonal matrix, and  $L_k$  be the corresponding Laplacian matrix.

##### 3.2.2. Using the subspace trick

Ideally, we want to map all the points belonging to the same class ( $c_k$  or  $\bar{c}_k$ ) into a single point. Since we only have a few labeled points for each class, the best we can do is to represent labeled points belonging to  $c_k$  by one 1-dimensional vector, represent other labeled points by another vector, and map similar points as closely as possible. Let  $\mathbf{y} = [y_1, \dots, y_m]^T$  be the result of  $k$ -th map. We have the following objective function:

$$\mathbf{y}_{opt} = \operatorname{argmin}_{\mathbf{y}} \sum_{ij} (y_i - y_j)^2 (W_k)_{ij} = \operatorname{argmin}_{\mathbf{y}} \mathbf{y}^T L_k \mathbf{y} \tag{7}$$

with constraints

$$\begin{cases} y_i = y_j & \text{if } \mathbf{x}_i, \mathbf{x}_j \in l_k \\ y_i = y_j & \text{if } \mathbf{x}_i \in l_a, \mathbf{x}_j \in l_b, a \neq k, \text{ and } b \neq k \\ y_i \neq y_j & \text{if } \mathbf{x}_i \in l_k, \mathbf{x}_j \in l_a \text{ and } a \neq k \\ \mathbf{y}^T D_k \mathbf{y} = 1 \end{cases}$$

Without loss of generality, assume the first  $m_1$  points  $d_1, \dots, d_{m_1}$  are labeled points belonging to  $c_1$ , the next  $m_2$  points  $d_{m_1+1}, \dots, d_{m_1+m_2}$  are labeled points belonging to  $c_2$ , and so on. All the rest points are unlabeled. In order to meet the above constraint, we introduce the label constraint matrix  $P_k \in \{0, \pm 1\}^{m \times (2+m-p)}$ , where  $p = m_1 + \dots + m_r$ . Each row  $i$  of  $P_k$  corresponds to point  $\mathbf{x}_i$ , and the matrix is represented as follows:

$$P_k = \begin{pmatrix} \mathbf{1}_{m_1} & -\mathbf{1}_{m_1} & \mathbf{0}_{m_1 \times (m-p)} \\ \vdots & \vdots & \vdots \\ \mathbf{1}_{m_{k-1}} & -\mathbf{1}_{m_{k-1}} & \mathbf{0}_{m_{k-1} \times (m-p)} \\ \mathbf{1}_{m_k} & \mathbf{1}_{m_k} & \mathbf{0}_{m_k \times (m-p)} \\ \mathbf{1}_{m_{k+1}} & -\mathbf{1}_{m_{k+1}} & \mathbf{0}_{m_{k+1} \times (m-p)} \\ \vdots & \vdots & \vdots \\ \mathbf{1}_{m_r} & -\mathbf{1}_{m_r} & \mathbf{0}_{m_r \times (m-p)} \\ \mathbf{1}_{(m-p)} & \mathbf{0}_{(m-p)} & I_{(m-p) \times (m-p)} \end{pmatrix}$$

Using the label constraint matrix  $P_k$ , we can map labeled points into two different points by introducing an auxiliary vector  $\mathbf{z}$  and equating:

$$\mathbf{y} = P_k \mathbf{z}$$

Substituting it into Eq. (7), we obtain the following optimization problem:

$$\mathbf{z}_{opt} = \operatorname{argmin}_{\mathbf{z}} \mathbf{z}^T P_k^T L_k P_k \mathbf{z} \tag{8}$$

with the constraint

$$\mathbf{z}^T P_k^T D_k P_k \mathbf{z} = 1$$

Note that three constraints of Eq. (7) are dropped. They are automatically guaranteed by equation  $\mathbf{y} = P_k \mathbf{z}$ , which we will explain later. By the Rayleigh–Ritz theorem [28], we know the solution of this problem is the smallest eigenvector of the following generalized eigenvector problem:

$$P_k^T L_k P_k \mathbf{z} = P_k^T D_k P_k \mathbf{z} \quad (9)$$

Let  $\mathbf{z}_0$  be the smallest eigenvalue of Eq. (9). We have the following proposition:

**Proposition 1.** *The smallest eigenvalue  $\mathbf{z}_0$  is 0 with eigenvector  $[1, 0, \dots, 0]^T$ , and the corresponding  $\mathbf{y}$  is the constant vector  $\mathbf{1}$ .*

**Proof.** It is easy to check that  $L_k$  is positive semi-definite and  $D_k$  is positive definite. Since the column vectors of  $P_k$  are independent, for any non-zero  $\mathbf{z}$ ,  $P_k \mathbf{z}$  is not a zero vector. Therefore,  $P_k^T L_k P_k$  is still positive semi-definite and  $P_k^T D_k P_k$  is positive definite. This implies that  $\geq 0$ . Actually, the smallest eigenvalue is 0 with eigenvector  $[1, 0, \dots, 0]^T$ , since

$$P_k^T L_k P_k [1, 0, \dots, 0]^T = P_k^T L_k \mathbf{1} = P_k^T \mathbf{0} = \mathbf{0}$$

The corresponding  $\mathbf{y} = P_k [1, 0, \dots, 0]^T = \mathbf{1}$ .  $\square$

Vector  $\mathbf{1}$  is useless, since all the data points have the same representations. So we choose the second smallest eigenvector  $\mathbf{z}_1$ . Let  $\mathbf{z}_1 = [z_1^1, z_2^1, \dots, z_{2+m-p}^1]^T$ , it is easy to check that

$$\mathbf{y} = \left[ \underbrace{(z_1^1 - z_2^1), \dots, (z_1^1 - z_2^1)}_{n_1 + \dots + n_{k-1}}, \underbrace{(z_1^1 + z_2^1), \dots, (z_1^1 + z_2^1)}_{n_k}, \right. \\ \left. \underbrace{(z_1^1 - z_2^1), \dots, (z_1^1 - z_2^1)}_{n_{k+1} + \dots + n_r}, \underbrace{(z_1^1 + z_3^1), \dots, (z_1^1 + z_{2+m-p}^1)} \right]^T$$

So labeled points belongs to  $c_k$  are represented by  $z_1^1 + z_2^1$ , and other labeled points are represented by  $z_1^1 - z_2^1$ . Since points belonging to same class are usually similar to one another, then in theory unlabeled points belonging to  $c_k$  will be mapped around  $z_1^1 + z_2^1$ , and other unlabeled points will be mapped around  $z_1^1 - z_2^1$ .

### 3.3. The algorithm

Given a set of data points  $\mathbf{x}_1, \dots, \mathbf{x}_m \in \mathbf{R}^n$  belonging to  $r$  classes and some labeled points for each class, CLE projects each points onto a  $r$ -dimensional discriminative space. It is performed as follows:

1. *Constructing the adjacency graph.* Let  $G$  denote a graph with  $m$  nodes, and the  $i$ -th node corresponds to the point  $\mathbf{x}_i$ . There are two variations [2]:
  - (1)  $\varepsilon$ - neighborhoods [parameter  $\varepsilon \in \mathbf{R}$ ]: Nodes  $i$  and  $j$  are connected by an edge if  $\|\mathbf{x}_i - \mathbf{x}_j\|^2 < \varepsilon$  where the norm is the usual Euclidean norm in  $\mathbf{R}^n$ .
  - (2)  $k$  nearest neighbors [parameter  $k \in \mathbf{N}$ ]: Nodes  $i$  and  $j$  are connected by an edge if  $\mathbf{x}_i$  is among  $k$  nearest neighbors of  $\mathbf{x}_j$  or  $\mathbf{x}_j$  is among  $k$  nearest neighbors of  $\mathbf{x}_i$ .
- *Choosing the weights.* There are many kinds of weighting methods [6]:
  - (1) 0-1 weighting,
  - (2) Gaussian kernel weighting,
  - (3) dot-product weighting, and
  - (4) polynomial kernel weighting.
3. *Mapping each data point into  $r$  different lines.* Each map  $i$  tries to separate points belonging to  $c_i$  from others by using the label information.

Suppose we are going to map each point into the  $k$ -th line. We use the following two steps to constrain the solution space of Laplacian Eigenmap (Section 3.2).

- (1) *Modifying the similarity matrix:*  
If both of two labeled points belong to  $c_k$ , the edge weight is increased. If neither of two labeled points belong to  $c_k$ , the edge weight is also increased. If one labeled point belongs to  $c_k$ , and the other labeled point does not, the edge weight is decreased.
- (2) *Using the subspace trick:*  
Firstly, we construct the label constraint matrix  $P_k$  for the  $k$ -th map. Secondly, we calculate the second smallest eigenvector  $\mathbf{z}_1$  of the generalized eigenvector problem (9). Then,  $\mathbf{y} = P_k \mathbf{z}_1$  gives the result of the  $k$ -th map.
4. *Combining the result of each mapping.* Denote the results of all the  $r$  maps as  $\mathbf{y}_1, \dots, \mathbf{y}_r$ . The embedding coordinate of point  $\mathbf{x}_i$  is given by the  $i$ -th row of  $Y = [\mathbf{y}_1, \dots, \mathbf{y}_r]$ .

After CLE, we get  $r$ -dimensional representations of the original data points. It is easy to check that labeled points belonging to the same class are merged together, labeled points belonging to different classes are separated, and similar points are close to one another. So, in this space, better classification or clustering performance can be obtained.

## 4. Experimental result

In this section, we perform document clustering using CLE to show the effectiveness of our algorithm. Document clustering has received a lot of attention as a fundamental tool for organization, summarization and retrieval of large volumes of text documents. Two standard document collections were used in the experiments: TDT2 and Reuters-21578.

### 4.1. Data corpora

We used the TDT2 and Reuters-21578 corpora as [6]. The TDT2 corpus<sup>1</sup> consists of documents collected from six sources (APW, NYT, VOA, PRI, CNN, and ABC) during 1998. It consists of 11,201 documents and 96 categories. In our experiments, we removed those documents belonging to two or more categories and used the largest 30 categories. This led to a data set with 9394 documents in 30 categories as described in Table 1. In this table, CluID means cluster's ID and DocNum means number of documents contained in the cluster.

Reuters-21578 corpus<sup>2</sup> contains 21,578 documents in 135 categories. In our experiments, we excluded those documents with multiple labels, and chose the largest 30 categories. It left us with 8067 documents in 30 categories as described in Table 2.

We removed the stop words, and represented each document as a term-frequency vector. Each document vector is normalized to unit length.

### 4.2. Evaluation metric

The clustering performance is evaluated by comparing the label obtained from our clustering algorithm with that provided by the document corpus. Two metrics, the accuracy (AC) and the normalized mutual information metric ( $\overline{MI}$ ), are used to measure the clustering performance [40,6]. Given a document  $d_i$ , let  $p_i$  and

<sup>1</sup> Nist Topic Detection and Tracking corpus is at <http://www.nist.gov/speech/tests/tdt/1998/>

<sup>2</sup> Reuters-21578 corpus is at <http://www.daviddlewis.com/resources/testcollections/reuters21578/>

3.  $k$ -means after LSI (LSI),
4.  $k$ -means after Laplacian Eigenmaps (LE),
5.  $k$ -means after semi-Laplacian Eigenmap (Semi-LE).

Since  $k$ -means algorithm can only find local minimum, and is sensitive to initial points. When we need to perform  $k$ -means, we apply it 10 times with different start points and the best result in

$q_i$  be the obtained cluster label and the label provided by the corpus, respectively. The AC is defined as follows:

$$AC = \frac{\sum_{i=1}^m \delta(q_i, \text{map}(p_i))}{m} \quad (10)$$

where  $m$  is the total number of documents,  $\delta(x,y)$  is the delta function that equals one if  $x=y$  and equals zero otherwise, and  $\text{map}(p_i)$  is the permutation mapping function that map each cluster label  $p_i$  to the equivalent label from the data corpus. The best mapping can be found by using the Kuhn–Munkres algorithm [27].

Let  $C$  denote the set of clusters provided by the document corpus and  $C'$  obtained from our algorithm. Their mutual information metric  $MI(C, C')$  is defined as following:

$$MI(C, C') = \sum_{c_i \in C, c'_j \in C'} p(c_i, c'_j) \cdot \log_2 \frac{p(c_i, c'_j)}{p(c_i) \cdot p(c'_j)}, \quad (11)$$

where  $p(c_i)$  and  $p(c'_j)$  are the probabilities that a document arbitrarily selected from the corpus belongs to the clusters  $c_i$  and  $c'_j$ , respectively, and  $p(c_i, c'_j)$  is the joint probability that the arbitrarily selected document belongs to the clusters  $c_i$  as well as  $c'_j$  at the same time. In our experiments, we use the normalized mutual information  $\overline{MI}$  as follows:

$$\overline{MI} = \frac{MI(C, C')}{\max(H(C), H(C'))} \quad (12)$$

where  $H(C)$  and  $H(C')$  are the entropies of  $C$  and  $C'$ , respectively. It is easy to check that  $\overline{MI}$  takes values between 0 and 1.

#### 4.3. Clustering results

To demonstrate how our method improves the performance of document clustering, we compared it with the following five methods:

1.  $k$ -means on original term-document matrix ( $k$ -means),
2. clustering based on Non-negative Matrix Factorization (NMF-NCW, [40]),

**Table 3**  
Performance comparison on TDT2 corpus.

<i>l</i>	Accuracy											
	<i>k</i> -Means	LSI	NMF-NCW	LE	3%		5%		7%		9%	
					Semi-LE	CLE	Semi-LE	CLE	Semi-LE	CLE	Semi-LE	CLE
2	0.9328	0.9092	0.9827	0.9987	0.9987	0.9987	0.9988	0.9988	0.9988	0.9989	0.9989	0.9989
3	0.8953	0.8546	0.9379	0.9956	0.9957	0.9964	0.9958	0.9963	0.9962	0.9966	0.9959	0.9961
4	0.8547	0.7580	0.9268	0.9910	0.9951	0.9948	0.9951	0.9945	0.9955	0.9935	0.9959	0.9948
5	0.8392	0.7686	0.9229	0.9907	0.9841	0.9885	0.9874	0.9911	0.9887	0.9913	0.9890	0.9911
6	0.8303	0.7538	0.9407	0.9764	0.9786	0.9782	0.9768	0.9807	0.9804	0.9824	0.9749	0.9827
Ave.	0.8704	0.8088	0.9422	0.9905	0.9904	0.9913	0.9908	0.9923	0.9919	0.9925	0.9909	0.9927
Mutual information												
2	0.8487	0.7645	0.9451	0.9765	0.9771	0.9777	0.9780	0.9788	0.9785	0.9792	0.9796	0.9809
3	0.7980	0.7388	0.8695	0.9626	0.9638	0.9683	0.9649	0.9684	0.9677	0.9702	0.9660	0.9675
4	0.7809	0.6853	0.8819	0.9542	0.9695	0.9682	0.9704	0.9684	0.9713	0.9673	0.9731	0.9696
5	0.7803	0.7040	0.8777	0.9563	0.9496	0.9552	0.9569	0.9614	0.9584	0.9621	0.9599	0.9628
6	0.8008	0.7137	0.8937	0.9379	0.9402	0.9427	0.9423	0.9493	0.9451	0.9513	0.9403	0.9528
Ave.	0.8018	0.7213	0.8936	0.9575	0.9600	0.9624	0.9625	0.9652	0.9642	0.9660	0.9638	0.9667

**Table 4**  
Performance comparison on Reuters corpus.

<i>l</i>	Accuracy											
	<i>k</i> -Means	LSI	NMF-NCW	LE	3%		5%		7%		9%	
					Semi-LE	CLE	Semi-LE	CLE	Semi-LE	CLE	Semi-LE	CLE
2	0.7486	0.7348	0.8689	0.8644	0.8872	0.9016	0.9007	0.9056	0.9051	0.9141	0.9127	0.9302
3	0.6434	0.6297	0.7247	0.7996	0.7996	0.8024	0.8173	0.8272	0.8293	0.8447	0.8456	0.8595
4	0.6896	0.6609	0.7635	0.7394	0.7492	0.7552	0.7706	0.7648	0.7849	0.7838	0.8013	0.8143
5	0.5328	0.5151	0.6539	0.7203	0.7495	0.7493	0.7697	0.7763	0.7912	0.7982	0.7985	0.8297
6	0.4921	0.4603	0.6029	0.6340	0.6493	0.6568	0.6665	0.7024	0.6941	0.7142	0.7232	0.7477
Ave.	0.6213	0.6002	0.7228	0.7515	0.7669	0.7731	0.7850	0.7952	0.8009	0.8110	0.8163	0.8363
Mutual information												
2	0.3095	0.2728	0.5242	0.5240	0.5581	0.5865	0.5920	0.6023	0.5962	0.6143	0.6192	0.6613
3	0.3867	0.3528	0.4266	0.5381	0.5397	0.5413	0.5643	0.5748	0.5859	0.6127	0.6087	0.6258
4	0.4691	0.4485	0.5119	0.5102	0.5094	0.5045	0.5229	0.5257	0.5343	0.5482	0.5529	0.5841
5	0.3726	0.3485	0.4225	0.5042	0.5171	0.5116	0.5395	0.5523	0.5573	0.5767	0.5627	0.6254
6	0.3679	0.3365	0.4107	0.4498	0.4636	0.4618	0.4785	0.5087	0.4993	0.5267	0.5152	0.5576
Ave.	0.3811	0.3518	0.4592	0.5053	0.5176	0.5211	0.5395	0.5528	0.5546	0.5757	0.5718	0.6108

document space again. Similarly, when the percentage of labeled documents is 3% or more, CLE outperforms both LE and Semi-LE on average. But this time the improvement is much more obvious. With 9% labeled documents, the average AC of CLE is about 2% point higher than that of Semi-LE and 8.5% point higher than LE; the average  $\overline{MI}$  is about 4% point higher than of Semi-LE and 10.5% point higher than of LE.

Fig. 1 shows the accuracy of the six algorithms with different percentages of labeled documents and different cluster number on Reuters corpus. Apparently, the performance of Semi-LE and CLE becomes better as the percentage of labeled documents increases. More important, the improvement of CLE compared with Semi-LE becomes more obvious as the percentage of labeled documents  $e$  or the number of clusters  $r$  increases.

## 5. Conclusion and future work

In this paper, we propose a novel semi-supervised dimensionality reduction algorithm called constrained Laplacian Eigenmap (CLE).

CLE aims to find the projection which respects the intrinsic geometrical structure inferred from all the data points, and also consists with labels. Document clustering experiments on TDT2 and Reuters-21578 corpora have shown that: when the label information is not scarce, CLE performs much better than the Semi-LE which incorporates label information through modifying the similarity matrix, and other unsupervised methods ( $k$ -means, LSI, NMF, LE). The advantage of CLE becomes more obvious as the percentage of labeled documents or the number of clusters increases.

Dimensionality reduction is widely used in face recognition. Eigenface [38], Fisherface [1], and Laplacianface [19,9] are three state-of-the-art face recognition techniques based on principal component analysis (PCA), linear discriminant analysis (LDA) and locality preserving projection (LPP), respectively. Compared with PCA and LDA, CLE can discover nonlinear manifolds. LPP seeks to preserve the intrinsic geometry of the data, but it is unsupervised. Previous studies have shown that face images are possibly reside on a nonlinear submanifold. Thus, CLE is very suitable for semi-supervised face recognition, which will be investigated in our future work.

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## References

- [1] P. Belhumeur, J. Hespanha, D. Kriegman, Eigenfaces vs. Fisherfaces: recognition using class specific linear projection, *IEEE Transactions on Pattern Analysis and Machine Intelligence* 19 (7) (1997) 711–720.
- [2] M. Belkin, P. Niyogi, Laplacian Eigenmaps and spectral techniques for embedding and clustering, in: *Advances in Neural Information Processing Systems*, vol. 14, MIT Press, Cambridge, MA, 2002, pp. 585–591.
- [3] M. Belkin, P. Niyogi, Towards a theoretical foundation for Laplacian-based manifold methods, in: *COLT '05: Proceedings of the 18th Annual Conference on Learning Theory*, , 2005, pp. 486–500.
- [4] T.D. Bie, J.A.K. Suykens, B.D. Moor, Learning from general label constraints, in: *SPR '04: Proceedings of IAPR International Workshop on Statistical Pattern Recognition*, 2004, pp. 671–679.
- [5] C.M. Bishop, *Pattern Recognition and Machine Learning (Information Science and Statistics)*, Springer, Berlin, 2007.
- [6] D. Cai, X. He, J. Han, Document clustering using locality preserving indexing, *IEEE Transactions on Knowledge and Data Engineering* 17 (12) (2005) 1624–1637.
- [7] D. Cai, X. He, J. Han, Semi-supervised discriminant analysis, in: *ICCV '07: IEEE 11th International Conference on Computer Vision*, , October 2007, pp. 1–7.
- [8] D. Cai, X. He, J. Han, Srda: an efficient algorithm for large-scale discriminant analysis, *IEEE Transactions on Knowledge and Data Engineering* 20 (1) (2008) 12.
- [9] D. Cai, X. He, J. Han, H.-J. Zhang, Orthogonal Laplacianfaces for face recognition, *IEEE Transactions on Image Processing* 15 (11) (2006) 3608–3614.
- [10] D. Cai, X. He, X. Wu, J. Han, Non-negative matrix factorization on manifold, in: *ICDM '08: Proceedings of the 8th IEEE International Conference on Data Mining*, IEEE Computer Society, Wao(ociaWao mage)-puterp2(D6Image)Cao.ma8(USAN

- [20] X. Ji, W. Xu, Document clustering with prior knowledge, in: SIGIR '06: Proceedings of the 29th Annual International ACM SIGIR Conference on Research and Development in Information Retrieval, ACM, New York, NY, USA, 2006, pp. 405–412.
- [21] S.D. Kamvar, D. Klein, C.D. Manning, Spectral learning, in: IJCAI '03: Proceedings of the 18th International Joint Conference on Artificial Intelligence, , 2003, pp. 561–566.
- [22] D.D. Lee, H.S. Seung, Learning the parts of objects by non-negative matrix factorization, *Nature* 401 (6755) (1999) 788–791.
- [23] D.D. Lee, H.S. Seung, Algorithms for non-negative matrix factorization, in: *Advances in Neural Information Processing Systems*, vol. 13, MIT Press, Cambridge, MA, 2001, pp. 556–562.
- [24] X. Li, S. Lin, S. Yan, D. Xu, Discriminant locally linear embedding with high-order tensor data, *IEEE Transactions on Systems, Man, and Cybernetics, Part B* 38 (2) (2008) 342–352.
- [25] C.-J. Lin, Projected gradient methods for nonnegative matrix factorization, *Neural Computation* 19 (10) (2007) 2756–2779.
- [26] Y.-Y. Lin, T.-L. Liu, H.-T. Chen, Semantic manifold learning for image retrieval, in: *MULTIMEDIA '05: Proceedings of the 13th Annual ACM International Conference on Multimedia*, ACM, New York, NY, USA, 2005, pp. 249–258.
- [27] L. Lovász, M.D. Plummer, *Matching Theory*, North-Holland, Amsterdam, 1986.
- [28] H. Lütkepohl, *Handbook of Matrices*, Wiley, Chichester, 1997.
- [29] A.Y. Ng, M.I. Jordan, Y. Weiss, On spectral clustering: analysis and an algorithm, in: *Advances in Neural Information Processing Systems*, vol. 14, MIT Press, Cambridge, MA, 2002, pp. 849–856.
- [30] Y. Pang, D. Tao, Y. Yuan, X. Li, Binary two-dimensional pca, *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics* 38 (4) (2008) 1176–1180.
- [31] Y. Pang, Y. Yuan, X. Li, Iterative subspace analysis based on feature line distance, *IEEE Transactions on Image Processing* 18 (4) (2009) 903–907.
- [32] S.T. Roweis, L.K. Saul, Nonlinear dimensionality reduction by locally linear embedding, *Science* 290 (5500) (2000) 2323–2326.
- [33] D. Tao, X. Li, W. Hu, S. Maybank, X. Wu, Supervised tensor learning, in: *ICDM '05: Proceedings of the 5th IEEE International Conference on Data Mining*, IEEE Computer Society, Washington, DC, USA, 2005, pp. 450–457.
- [34] D. Tao, X. Li, X. Wu, S. Maybank, Geometric mean for subspace selection, *IEEE Transactions on Pattern Analysis and Machine Intelligence* 31 (2) (2009) 260–274.
- [35] D. Tao, X. Li, X. Wu, S.J. Maybank, General tensor discriminant analysis and gabor features for gait recognition, *IEEE Transactions on Pattern Analysis and Machine Intelligence* 29 (10) (2007) 1700–1715.
- [36] D. Tao, X. Li, X. Wu, S.J. Maybank, Tensor rank one discriminant analysis—a convergent method for discriminative multilinear subspace selection, *Neurocomputing* 71 (10–12) (2008) 1866–1882.
- [37] J.B. Tenenbaum, V. de Silva, J.C. Langford, A global geometric framework for nonlinear dimensionality reduction, *Science* 290 (5500) (2000) 2319–2323.
- [38] M.A. Turk, A.P. Pentland, Face recognition using eigenfaces, in: *CVPR '91: Proceedings of 1991 IEEE Computer Society Conference on Computer Vision and Pattern Recognition*, , 1991, pp. 586–591.
- [39] D. Xu, S. Yan, D. Tao, L. Zhang, X. Li, H.-J. Zhang, Human gait recognition with matrix representation, *IEEE Transactions on Circuits and Systems for Video Technology* 16 (7) (2006) 896–903.
- [40] W. Xu, X. Liu, Y. Gong, Document clustering based on non-negative matrix factorization, in: *SIGIR '03: Proceedings of the 26th Annual International ACM SIGIR Conference on Research and Development in Information Retrieval*, ACM, New York, NY, USA, 2003, pp. 267–273.
- [41] S. Yan, D. Xu, B. Zhang, H. jiang Zhang, Graph embedding: a general framework for dimensionality reduction, in: *CVPR 05': Proceedings of the Internal Conference on Computer Vision and Pattern Recognition*, 2005, pp. 830–837.
- [42] J. Yang, S. Yang, Y. Fu, X. Li, T. Huang, Non-negative graph embedding, in: *CVPR '08: Proceedings of the 2008 IEEE Computer Society Conference on Computer Vision and Pattern Recognition*, IEEE Computer Society, Washington, DC, USA, June 2008.
- [43] Y. Yuan, Y. Pang, Discriminant adaptive edge weights for graph embedding, in: *ICASSP '08: IEEE International Conference on Acoustics, Speech and Signal Processing*, 2008, pp. 1993–1996.
- [44] T. Zhang, D. Tao, X. Li, J. Yang, Patch alignment for dimensionality reduction, *IEEE Transactions on Knowledge and Data Engineering* 21 (9) (2009) 1299–1313.
- [45] T. Zhang, D. Tao, J. Yang, Discriminative locality alignment, in: *ECCV '08: Proceedings of the 10th European Conference on Computer Vision*, Springer, Berlin, Heidelberg, 2008, pp. 725–738.



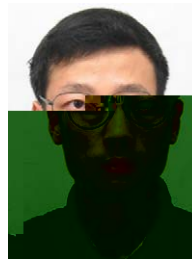
**Chun Chen** received the BS degree in Mathematics from Xiamen University, China, in 1981, and his MS and Ph.D. degrees in Computer Science from Zhejiang University, China, in 1984 and 1990, respectively. He is a professor in College of Computer Science, Zhejiang University. His research interests include information retrieval, data mining, computer vision, computer graphics and embedded technology.



**Lijun Zhang** received the BS degree in Computer Science from Zhejiang University, China, in 2007. He is currently a candidate for a Ph.D. degree in Computer Science from Zhejiang University. His research interests include machine learning, information retrieval, and data mining.



**Jiajun Bu** received the BS and Ph.D. degrees in Computer Science from Zhejiang University, China, in 1995 and 2000, respectively. He is a professor in College of Computer Science, Zhejiang University. His research interests include embedded system, data mining, information retrieval and mobile database.



**Can Wang** received the BS degree in Economics, MS and Ph.D. degrees in Computer Science from Zhejiang University, China, in 1995, 2003 and 2009, respectively. He is currently a faculty member in College of Computer Science at Zhejiang University. His research interests include information retrieval, data mining and machine learning.



**Wei Chen** received the BS degree in Computer Science from Zhejiang University, China, in 2005. He is currently a candidate for a Ph.D. degree in Computer Science at Zhejiang University. His research interests include mobile and ubiquitous information systems, web information filtering and retrieval, and accessible computing.