# Scalable Demand-Aware Recommendation

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## Abstract

Recommendation for e-commerce with a mix of durable and nondurable goods has characteristics that distinguish it from the well-studied media recommendation problem. The demand for items is a combined effect of *form utility* and *time utility*, i.e., a product must both be intrinsically appealing to a consumer and the time must be right for purchase. In particular for durable goods, time utility is a function of inter-purchase duration within product category because consumers are unlikely to purchase two items in the same category in close temporal succession. Moreover, purchase data, in contrast to rating data, is implicit with non-purchases not necessarily indicating dislike. Together, these issues give rise to the positive-unlabeled demand-aware recommendation problem that we pose via joint low-rank tensor completion and product category inter-purchase duration vector estimation. We further relax this problem and propose a highly scalable alternating minimization approach with which we can solve problems with millions of users and millions of items in a single thread. We also show superior prediction accuracies on multiple real-world datasets.

## 1 Introduction

E-commerce recommender systems aim to present items with high utility to the consumers [18]. Utility may be decomposed into *form utility*: the item is desired as it is manifested, and *time utility*: the item is desired at the given point in time [28]; recommender systems should take both types of utility into account. Economists define items to be either *durable goods*

2.

forms the basis for our modeling approach. A variety of time-aware recommender systems have been proposed to exploit time information, but none of them explicitly consider the notion of time utility derived from inter-purchase durations in item categories. Much of the PU learning literature is focused on the binary classification problem, e.g. [20, 9], whereas we are in the collaborative filtering setting. For the papers that do examine collaborative filtering with PU learning or learning with implicit feedback [14, 23, 2, 32], they mainly focus on media recommendation and overlook users' demands, thus are not suitable for durable goods recommendation.

Temporal aspects of the recommendation problem have been examined in a few ways: as part of the cold-start problem [3], to capture dynamics in interests or ratings over time [17], and as part of the context in context-aware recommenders [1]. However, the problem we address in this paper is different from all of those aspects, and in fact could be combined with the other aspects in future solutions. To the best of our knowledge, there is no existing work that tries to take inter-purchase durations into account to better time recommendations as we do herein.

## 3 Positive-Unlabeled Demand-Aware Recommendation

Throughout the paper, we use boldface Euler script letters, boldface capital letters, and boldface lower-case letters to denote tensors (e.g., $\boldsymbol{\mathcal{A}}$), matrices (e.g., $\mathbf{A}$) and vectors (e.g., $\mathbf{a}$), respectively. Scalars such as entries of tensors, matrices, and vectors are denoted by lowercase letters, e.g., $a$. In particular, the $(i, j, k)$ entry of a third-order tensor $\boldsymbol{\mathcal{A}}$ is denoted by $a_{ijk}$.

Given a set of $m$ users, $n$ items, and $l$ time slots, we construct a third-order binary tensor $\boldsymbol{\mathcal{P}} \in \{0, 1\}^{m \times n \times l}$ to represent the purchase history. Specifically, entry $p_{ijk} = 1$ indicates that user $i$ has purchased item $j$ in time slot $k$. We denote $\|\boldsymbol{\mathcal{P}}\|_0$ as the number of nonzero entries in tensor $\boldsymbol{\mathcal{P}}$. Since $\boldsymbol{\mathcal{P}}$ is usually very sparse, we have $\|\boldsymbol{\mathcal{P}}\|_0 \ll mnl$. Also, we assume that the $n$ items belong to $r$ item categories, with items in each category sharing similar inter-purchase durations.[3] We use an $n$-dimensional vector $\mathbf{c} \in \{1, 2, \ldots, r\}^n$ to represent the category membership of each item. Given $\boldsymbol{\mathcal{P}}$ and $\mathbf{c}$, we further generate a tensor $\boldsymbol{\mathcal{T}} \in \mathbb{R}^{m \times r \times l}$ where $t_{ic_jk}$ denotes the number of time slots between user $i$'s most recent purchase within item category $c_j$ until time $k$. If user $i$ has not purchased within item category $c_j$ until time $k$, $t_{ic_jk}$ is set to $+\infty$.

### 3.1 Inferring Purchase Intentions from Users' Purchase Histories

In this work, we formulate users' utility as a combined effect of form utility and time utility. To this end, we use an underlying third-order tensor $\boldsymbol{\mathcal{X}} \in \mathbb{R}^{m \times n \times l}$ to quantify form utility. In addition, we employ a non-negative vector $\mathbf{d} \in \mathbb{R}^r_+$ to measure the underlying inter-purchase duration times of the $r$

Note that the positive entries of $\mathcal{Y}$ denote high purchase intentions, while the positive entries of $\mathcal{P}$ denote actual purchases. Generally speaking, a purchase only happens when the utility is high, but a high utility does not necessarily lead to a purchase. This observation allows us to link the binary tensors $\mathcal{P}$ and $\mathcal{Y}$: $\mathcal{P}$ is generated by a one-sided sampling process that only reveals a subset of $\mathcal{Y}$'s positive entries. Given this observation, we follow [13] and include a label-dependent loss [26] trading the relative cost of positive and unlabeled samples:

$$\mathcal{L}(\mathcal{X}; \mathcal{P}) = \sum_{ijk:\, p_{ijk}=1} \max[1 - (x_{ijk} - \max(0; d_{c_j} - t_{ic_jk})); 0]^2 + (1 - \quad) \sum_{ijk:\, p_{ijk}=0} l(x_{ijk}; 0);$$

where $l(x; c) = (x - c)^2$ denotes the squared loss.

In addition, the form utility tensor $\mathcal{X}$ should be of low-rank to capture temporal dynamics of users' interests, which are generally believed to be dictated by a small number of latent factors [22].

By combining asymmetric sampling and the low-rank property together, we jointly recover the tensor $\mathcal{X}$ and the inter-purchase duration vector $\mathbf{d}$ by solving the following tensor nuclear norm minimization (TNNM) problem:

$$\min_{\mathcal{X} \in \mathbb{R}^{ijc}}$$

## 4 Optimization

Although the learning problem has been relaxed, optimizing (4) is still very challenging for two main reasons: (i) the objective is highly non-smooth with nested hinge losses, and (ii) it contains $mnl$ terms, and a naive optimization algorithm will take at least $O(mnl)$ time.

To address these challenges, we adopt an alternating minimization scheme that iteratively fixes one of $\mathbf{d}$ and $\mathbf{X}$ and minimizes with respect to the other. Specifically, we propose an extremely efficient optimization algorithm by effectively exploring the sparse structure of the tensor $\mathcal{P}$ and low-rank structure of the matrix $\mathbf{X}$. We show that (i) the problem (4) can be solved within $O(\|\mathcal{P}\|_0(k + \log(\|\mathcal{P}\|_0)) + (n+m)k^2)$ time, where $k$ is the rank of $\mathbf{X}$, and (ii) the algorithm converges to the critical points of $f(\mathbf{X}, \mathbf{d})$. In the following, we provide a sketch of the algorithm. The detailed description can be found in the supplementary material.

### 4.1 Update $\mathbf{d}$

When $\mathbf{X}$ is fixed, the optimization problem with respect to $\mathbf{d}$ can be written as:

$$\min_{\mathbf{d}} \sum_{ijk:\, p_{ijk}=1} \left( \max\left(1 - (x_{ij} - \max(0, d_{c_j} - t_{ic_jk})), 0\right) \right)^2 := g(\mathbf{d}) := \sum_{ijk:\, p_{ijk}=1} g_{ijk}(d_{c_j}). \quad (5)$$

Problem (5) is non-trivial to solve since it involves nested hinge losses. Fortunately, by carefully analyzing the value of each term $g_{ijk}(d_{c_j})$, we can show that

$$g_{ijk}(d_{c_j}) = \begin{cases} \max(1 - x_{ij}, 0)^2, & \text{if } d_{c_j} \le t_{ic_jk} + \max(x_{ij} - 1, 0) \\ (1 - (x_{ij} - d_{c_j} + t_{ic_jk}))^2, & \text{if } d_{c_j} > t_{ic_jk} + \max(x_{ij} - 1, 0). \end{cases}$$

For notational simplicity, we let $s_{ijk} = t_{ic_jk} + \max(x_{ij} - 1, 0)$ for all triplets $(i, j, k)$ satisfying $p_{ijk} = 1$. Now we can focus on each category : for each , we collect the set $Q = \{(i, j, k) \mid p_{ijk} = 1 \text{ and } c_j = \}$ and calculate the corresponding $s_{ijk}$s. We then sort $s_{ijk}$s such that $s_{(i_1 j_1 k_1)} \le \cdots \le s_{(i_{jQ} j_{jQ} k_{jQ})}$. For each interval $[s_{(i_q j_q k_q)}, s_{(i_{q+1} j_{q+1} k_{q+1})}]$, the function is quadratic, thus can be solved in a closed form. Therefore, by scanning the solution regions from left to right according to the sorted $s$ values, and maintaining some intermediate computed variables, we are able to find the optimal solution, as summarized by the following lemma:

**Lemma 1.** *The subproblem* (5) *is convex with respect to* $\mathbf{d}$ *and can be solved exactly in* $O(\|\mathcal{P}\|_0 \log(\|\mathcal{P}\|_0))$*, where* $\|\mathcal{P}\|_0$ *is the number of nonzero elements in tensor* $\mathcal{P}$*.*

Therefore, we can efficiently update $\mathbf{d}$ since $\mathcal{P}$ is a very sparse tensor with only a small number of nonzero elements.

### 4.2 Update $\mathbf{X}$

By defining

$$a_{ijk} = \begin{cases} 1 + \max(0, d_{c_j} - t_{ic_jk}), & \text{if } p_{ijk} = 1 \\ 0, & \text{otherwise} \end{cases}$$

the subproblem with respect to $\mathbf{X}$ can be written as

$$\min_{\mathbf{X} \in \mathbb{R}^{m \; n}} h(\mathbf{X}) + \|\mathbf{X}\|_* \text{ where } h(\mathbf{X}) := \sum_{ijk:\, p_{ijk}=1} \max(a_{ijk} - x_{ij}, 0)^2 + (1 - ) \sum_{ijk:\, p_{ijk}=0} x_{ij}^2 : \quad (6)$$

Since there are $O(mnl)$ terms in the objective function, a naive implementation will take at least $O(mnl)$ time, which is computationally infeasible when the data is large. To address this issue, We use proximal gradient descent to solve the problem. At each iteration, $\mathbf{X}$ is updated by

$$\mathbf{X} \leftarrow S\ (\mathbf{X} - \ \nabla h(\mathbf{X})); \quad (7)$$

where $S\ (\cdot)$ is the soft-thresholding operator for singular values.[5]

[5] If $\mathbf{X}$ has the singular value decomposition $\mathbf{X} = \mathbf{U}\ \ \mathbf{V}^T$, then $S\ (\mathbf{X}) = \mathbf{U}(\ \ \lambda I)_+ \mathbf{V}^T$ where $a_+ = \max(0, a)$.

Table 1: CPU time for solving problem (4) with different number of purchase records

| $m$ (# users) | $n$ (# items) | $l$ (# time slots) | $\|\mathcal{P}\|_0$ | $k$ | CPU Time (in seconds) |
|---|---|---|---|---|---|
| 1,000,000 | 1,000,000 | 1,000 | 11,112,400 | 10 | 595 |
| 1,000,000 | 1,000,000 | 1,000 | 43,106,100 | 10 | 1,791 |
| 1,000,000 | 1,000,000 | 1,000 | 166,478,000 | 10 | 6,496 |

In order to efficiently compute the top singular vectors of $\mathbf{X} - \nabla h(\mathbf{X})$, we rewrite it as

$$\mathbf{X} - \nabla h(\mathbf{X}) = [1 - 2(1 - )/] \mathbf{X} + \left(2(1 - ) \sum_{ijk:\, p_{ijk}=1} x_{ij} - 2 \sum_{ijk:\, p_{ijk}=1} \max(a_{ijk} - x_{ij}; 0)\right):$$

$$= f_a(\mathbf{X}) + f_b(\mathbf{X}):$$

Since $f_a(\mathbf{X})$ is of low-rank and $f_b(\mathbf{X})$ is sparse, multiplying $(\mathbf{X} - \nabla h(\mathbf{X}))$ with a skinny $m$ by $k$ matrix can be computed in $O(nk^2 + mk^2 + \|\mathcal{P}\|_0 k)$ time. As shown in [12], each iteration of proximal gradient descent for nuclear norm minimization only requires a fixed number of iterations of randomized SVD (or equivalently, power iterations) using the warm start strategy, thus we have the following lemma.

**Lemma 2.** *A proximal gradient descent algorithm can be applied to solve problem* (6) *within* $O(nk^2T + mk^2T + \|\mathcal{P}\|_0 kT)$ *time, where* $T$ *is the number of iterations.*

We note that the algorithm is guaranteed to converge to the true solution. This is because when we apply a fixed number of iterations to update $\mathbf{X}$ via problem (7), it is equivalent to the "inexact gradient descent update" where each gradient is approximately computed and the approximation error is upper bounded by a constant between zero and one. Intuitively speaking, when the gradient converges to 0, the error will also converge to 0 at an even faster rate. See [12] for the detailed explanations.

### 4.3 Overall Algorithm

Combining the two subproblems together, the time complexity of each iteration of the proposed algorithm is:

$$O(\|\mathcal{P}\|_0 \log(\|\mathcal{P}\|_0) + nk^2T + mk^2T + \|\mathcal{P}\|_0 kT):$$

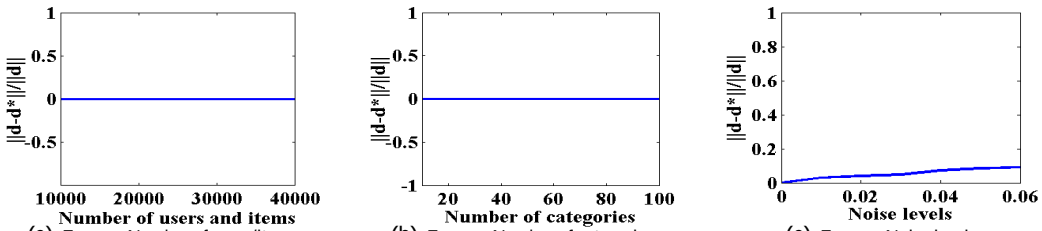
**Remark:** Since each user should make at least one purchase and each item should be purchased at least once to be included in $\mathcal{P}$, $n$ and $m$ are smaller than $\|\mathcal{P}\|_0$. Also, since $k$ and $T$ are usually very small, the time complexity to solve problem (4) is dominated by the term $\|\mathcal{P}\|_0$, which is a significant improvement over the naive approach with at least $O(mnl)$ complexity.

Since our problem has only two blocks $\mathbf{d}$; $\mathbf{X}$ and each subproblem is convex, our optimization algorithm is guaranteed to converge to a stationary point [11]. Indeed, it converges very fast in practice. As a concrete example, our experiment shows that it takes only 9 iterations to optimize a problem with 1 million users, 1 million items, and more than 166 million purchase records.

## 5 Experiments

### 5.1 Experiment with Synthesized Data

We first conduct experiments with simulated data to verify that the proposed demand-aware recommendation algorithm is computationally efficient and robust to noise. To this end, we first construct a low-rank matrix $\mathbf{X} = \mathbf{W}\mathbf{H}^T$, where $\mathbf{W} \in \mathbb{R}^{m \times 10}$ and $\mathbf{H} \in \mathbb{R}^{n \times 10}$ are random Gaussian matrices with entries drawn from $\mathcal{N}(1; 0.5)$, and then normalize $\mathbf{X}$ to the range of $[0; 1]$. We randomly assign all the $n$ items to $r$ categories, with their inter-purchase durations $\mathbf{d}$ equaling $[10; 20; \cdots; 10r]$. We then construct the high purchase intension set $= \{(i; j; k) \mid t_{ic_jk} \geq d_{c_j}$ and $x_{ij} \geq 0.5\}$, and sample a subset of its entries as the observed purchase records. We let $n = m$ and vary them in the range $\{10,000; 20,000; 30,000; 40,000\}$. We also vary $r$ in the range $\{10; 20; \cdots; 100\}$. Given the learned durations $\mathbf{d}^*$, we use $\|\mathbf{d} - \mathbf{d}^*\|_2 / \|\mathbf{d}\|_2$ to measure the prediction errors.




(a) Error vs Number of users/items (b) Error vs Number of categories (c) Error vs Noise levels

Figure 1: Prediction errors $\|\mathbf{d} - \mathbf{d}^*\|_2 = \|\mathbf{d}\|_2$ as a function of number of users, items, categories, and noise levels on synthetic datasets

**Accuracy** Figure 1(a) and 1(b) clearly show that the proposed algorithm can *perfectly* recover the underlying inter-purchase durations with varied numbers of users, items, and categories. To further evaluate the robustness of the proposed algorithm, we randomly flip some entries in tensor $\mathcal{P}$ from 0 to 1 to simulate the rare cases of purchasing two items in the same category in close temporal succession. Figure 1(c) shows that when the ratios of noisy entries are not large, the predicted durations $\hat{\mathbf{d}}$ are close enough to the true durations, thus verifying the robustness of the proposed algorithm.

**Scalability** To verify the scalability of the proposed algorithm, we fix the numbers of users and items to be 1 million, the number of time slots to be 1*;* 000, and vary the number of purchase records (i.e., $\|\mathcal{P}\|_0$). Table 1 summarizes the CPU time of solving problem (4) on an Intel Xeon 2.40 GHz server with 32 GB main memory. We observe that the proposed algorithm is extremely efficient, e.g., even with 1 million users, 1 million items, and more than 166 million purchase records, the running time of the proposed algorithm is less than 2 hours.

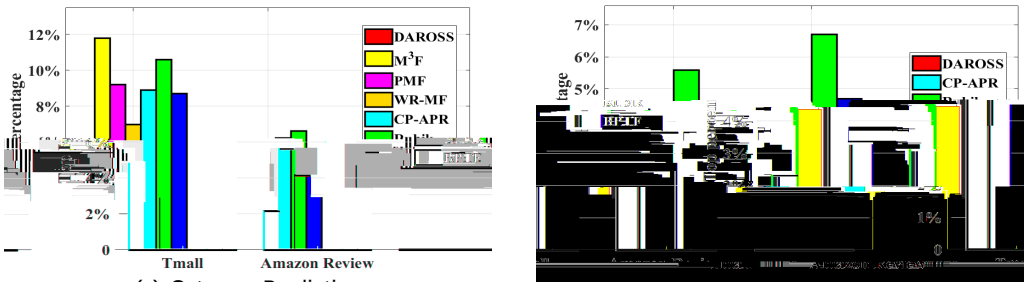
### 5.2 Experiment with Real-World Data

In the real-world experiments, we evaluate the proposed demand-aware recommendation algorithm by comparing it with the six state-of the-art recommendation methods: (a) $\mathbf{M^3F}$, maximum-margin matrix factorization [24], (b) **PMF**, probabilistic matrix factorization [25], (c) **WR-MF**, weighted regularized matrix factorization [14], (d) **CP-APR**, Candecomp-Parafac alternating Poisson regression [7], (e) **Rubik**, knowledge-guided tensor factorization and completion method [30], and (f) **BPTF**, Bayesian probabilistic tensor factorization [31]. Among them, M$^3$F and PMF are widely-used static collaborative filtering algorithms. We include these two algorithms as baselines to justify whether traditional collaborative filtering algorithms are suitable for general e-commerce recommendation involving both durable and nondurable goods. Since they require explicit ratings as inputs, we follow [2] to generate numerical ratings based on the frequencies of (user, item) consumption pairs. WR-MF is essentially the positive-unlabeled version of PMF and has shown to be very effective in modeling implicit feedback data. All the other three baselines, i.e., CP-APR, Rubik, and BPTF, are tensor-based methods that can consider time utility when making recommendations. We refer to the proposed recommendation algorithm as **Demand-Aware Recommender for One-Sided Sampling**, or **DAROSS** for short.

Our testbeds are two real-world datasets *Tmall*[6] and *Amazon Review*[7]. Since some of the baseline algorithms are not scalable enough, we first conduct experiments on their subsets and then on the full set of *Amazon Review*. In order to generate the subsets, we randomly sample 80 item categories for *TmaII dataset* and select the users who have purchased at least 3 items within these categories, leading to the purchase records of 377 users and 572 items. For *Amazon Review dataset*, we randomly select 300 users who have provided reviews to at least 5 item categories on Amazon.com. This leads to a total of 5*;* 111 items belonging to 11 categories. Time information for both datasets is provided in days, and we have 177 and 749 time slots for *TmaII* and *Amazon Review* subsets, respectively. The full *Amazon Review* dataset is significantly larger than its subset. After removing duplicate items, it contains more than 72 million product reviews from 19*:*8 million users and 7*:*7 million items that

[6] http://ijcai-15.org/index.php/repeat-buyers-prediction-competition

[7] http://jmcauley.ucsd.edu/data/amazon/




(a) Category Prediction (b) Purchase Time Prediction

Figure 2: Prediction performance on real-world datasets *Tmall* and *Amazon Review* subsets

Table 2: Estimated inter-review durations for Amazon Review subset

| Categories | Instant Video | Apps for Android | Automotive | Baby | Beauty | Digital Music | Grocery ... Food | Musical Instruments | Office Products | Patio ... Garden | Pet Supplies |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\mathbf{d}$ | 0 | 0 | 326 | 0 | 0 | 158 | 0 | 38 | 94 | 271 | 40 |

belong to 24 item categories. The collected reviews span a long range of time: from May 1996 to July 2014, which leads to 6*;* 639 time slots in total. Comparing to its subset, the full set is a much more challenging dataset both due to its much larger size and much higher sparsity, i.e., many reviewers only provided a few reviews, and many items were only reviewed a small number of times.

For each user, we randomly sample 90% of her purchase records as the training data, and use the remaining 10% as the test data. For each purchase record ($u$, $i$, $t$) in the test set, we evaluate all the algorithms on two tasks: (i) *category prediction*, and (ii) *purchase time prediction*. In the first task, we record the highest ranking of items that are within item $i$'s category among all items at time $t$. Since a purchase record ($u$, $i$, $t$) may suggest that in time slot $t$, user $u$ needed an item that share similar functionalities with item $i$, *category prediction* essentially checks whether the recommendation algorithms recognize this need. In the second task, we record the number of slots between the true purchase time $t$ and its nearest predicted purchase time within item $i$'s category. Ideally, good recommendations should have both small category rankings and small time errors. Thus we adopt the average top percentages, i.e., (*average category ranking*) / $n \times 100\%$ and (*average time error*) / $l \times 100\%$, as the evaluation metrics of category and purchase time prediction tasks, respectively. The algorithms M$^3$F, PMF, and WR-MF are excluded from the purchase time prediction task since they are static models that do not consider time information.

Figure 2 displays the predictive performance of the seven recommendation algorithms on *Tmall* and *Amazon Review* subsets. As expected, M$^3$F and PMF fail to deliver strong performance since they neither take into account users' demands, nor consider the positive-unlabeled nature of the data. This is verified by the performance of WR-MF: it significantly outperforms M$^3$F and PMF by considering the PU issue and obtains the second-best item prediction accuracy on both datasets (while being unable to provide a purchase time prediction). By taking into account both issues, our proposed algorithm DAROSS yields the best performance for both datasets and both tasks. Table 2 reports the *inter-review* durations of *Amazon Review* subset estimated by our algorithm. Although they may not perfectly reflect the true inter-purchase durations, the estimated durations clearly distinguish between durable good categories, e.g., *automotive*, *musical instruments*, and non-durable good categories, e.g., *instant video*, *apps*, and *food*. Indeed, the learned inter-purchase durations can also play an important role in applications more advanced than recommender systems, such as inventory management, operations management, and sales/marketing mechanisms. We do not report the estimated durations of *Tmall* herein since the item categories are anonymized in the dataset.

Finally, we conduct experiments on the full *Amazon Review* dataset. In this study, we replace *category prediction* with a more strict evaluation metric *item prediction* [8], which indicates the predicted ranking of item $i$ among all items at time $t$ for each purchase record ($u$, $i$, $t$) in the test set. Since most of our baseline algorithms fail to handle such a large dataset, we only obtain the predictive performance of three algorithms: DAROSS, WR-MF, and PMF. Note that for such a large dataset, prediction time instead of training time becomes the bottleneck: to evaluate average item rankings, we

need to compute the scores of all the 7.7 million items, thus is computationally inefficient. Therefore, we only sample a subset of items for each user and estimate the rankings of her purchased items. Using this evaluation method, the average item ranking percentages for DAROSS, WR-MF, and PMF are 16.7%, 27.3%, and 38.4%, respectively. In addition to superior performance, it only takes our algorithm 10 iterations and 1 hour to converge to a good solution. Since WR-MF and PMF

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