# **Multi-Objective Generalized Linear Bandits**

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 $\{1, a, b, b, a^{-1}\}$  @1a dat .ed .ot, a b @a1 baba-t c.c

## Abstract

I  $\frac{1}{4}$  a ef, e, d  $\frac{1}{4}$  e  $\frac{1}{4}$  - b ec<sub>4</sub> e bà d'<sub>4</sub> (MOB) r b e , e e a e at e r e e a e d e e c<sub>4</sub> 1 e-ar i a a d i d rece e are ard ec r c1 i K i e bec e. MOB a K 1 d ă real-rida i cari a areda i i erec de da i d di e, r r d . O f e f e f k d d, e e a l ca i , call ci tai ci te -ta d Kr a i f a cà de f e e e at i r -ce k d f, e e c, i red b , Ke f i r . T f e f i Kr a i, e a ca e eat ar eale ar haci te cetra da ente-re-ard K<sup>II</sup> he deta ed rear de (GLM). We ad the et it KPares re-ress e a ase releaŭer'erKrăce-ădde-elaĭ elalr Kritt Kree e ja dea sa -Ta ară, Kreilre-Ne, i se se jase de ara exer, ba ed 1 k & e 1 eke er c'i fide ce-b 1 d (UCB), c, c'i r cr  $\dot{a}_{1} = r_{1} = a_{1} + K_{1} + Pare_{4} + K_{1} + \dot{a}_{4} +$  $f_{a}$  for edal  $f_{a}$  at one a  $O(\sqrt{)}$ Pares re-res, cee fee er i ad ged e i Krises, e aseefee a a re-1, Kr i le beck e ci ke k a bà d' r ble. N ercale er de de 1 staselse ekkecter Kreitd.

## **1** Introduction

rid Meretar k le dia la ble i ke ee il alar.

Al a, ra e, d i KMAB i e i beccelar ar ed bà d', (MOMAB), r ed b Dr à à d N [2013], e e e e re ard e al i à ar a d', d i a e c, r i e ead Ka ca'ar a' e. I i e e i , d'Here, ar are-c ared acc rdi Pare, rder bee e e e re ard e c, r à d'i e ar i e re ard arei i Kerr i a, Kà i e ar are-called Pare, a ar , al Ki e ci i e e ar e e e e e a dard erc i e Pare, re re, i e e ear e e e c a e a be, e e e e e ard Ke e ear e a d'i a, Ke Pare, Ki Fe-a e e e ard Ke e ear e a d'i a, Ke Pare, Ki Fe-a e e e ard Ke e ear e a d'i a, Ke Pare, ki Fe-a e e e de i i i e e a ri e e e a ar a be e e e e e a e i e e a i Pare, a ar ba ed i e r cal be a ar a a e al a be. MOMAB a a e be real r'd a cai i e d'i e e e e a i becce, e a i e a d'i e i rec è da i e e (R dr e é 2012]. O e e e e a d'i e e ar i kr ai (ci e ) a ca i cal ci ai a ar i Kr ai (ci e ) a ca i e e e e i e e i e e i e e a e r fie i rec è da i e e i e e i e e a e r fie i rec è da i e e e e e i e e a e r fie i rec è da i e e i e a e r fie i rec e d'a i e e i e e e e e i e e a e r fie i rec e d'a i e e (L é e 2010], e e i red b MOMAB.

K er a r b b add at . T addre  $\frac{1}{4}$  b add at . T addre  $\frac{1}{4}$  b a r b er a d **K** f a **e** f - b er e - ar a c a b a d f 1 def a d **K** f a f a f - b er a - a c er ed real ab a f a f 1, b er a be e - a - e a d ed t 1 e - b er e - c 1 e - a b a d a [A ef, 2002; Dà  $\frac{1}{4}$ , 2008]. Cì creat, e de la ecirera cared la ar a a d' e i a ecari  $\in \mathbb{R}^d$  à d'Kr le a e Kolar, de tela e b i. R'ere ard ecar eral i à ar cì K bect e R'a e d'i ear de [Neder à d'Wedderb i , 1972] e la a

$$\mathbb{E}\begin{bmatrix}i \\ i\end{bmatrix} = \mu_i(\theta_i^\top), \quad = 1, \dots$$

k efe<sup>-i</sup>re-re-d<sup>-i</sup>re k e<sup>-j</sup>rc <sup>1</sup>d<sup>-i</sup> K , θ ,..., θ<sub>m</sub> areec<sub>i</sub>r K <sup>1</sup> <sup>1</sup> <sup>1</sup> c effice<sup>-i</sup>r, a d  $\mu$  ,...,  $\mu_m$  are-<sup>1</sup> Ki c<sub>i</sub> <sup>1</sup> . We-refe<sup>-i</sup>r Kr<sup>-1</sup>a<sub>i</sub> <sup>1</sup> a<sup>-1</sup>ra<sup>-1</sup>bec<sub>i</sub> e<sup>-j</sup> d<sup>-</sup>efa<sup>-</sup> ed<sup>-1</sup>i ear bà d<sup>-</sup><sub>i</sub> (MOGLB), <sup>k</sup> e<sup>-</sup> e<sup>-</sup> d<sup>-</sup>efa<sup>-</sup> à dc e<sup>-</sup> a derà e<sup>-</sup> Kr<sup>-1</sup>be<sup>-</sup>, e<sup>-</sup> a e<sup>-</sup> d<sup>-</sup>efa<sup>-</sup> à dc e<sup>-</sup> a derà e<sup>-</sup> Kr<sup>-</sup>be<sup>-</sup>, e<sup>-</sup> a e<sup>-</sup> d<sup>-</sup>efa<sup>-</sup> bà d<sup>-</sup> [A e<sup>-</sup>, 2002; Da<sup>-</sup> 4, 2008] a d<sup>-1</sup>i e<sup>-</sup> e<sup>-</sup> a ic<sup>-1</sup>i ear a<sup>-</sup> a<sup>-</sup> 1 der bi ar Kedbac [2] a<sup>-</sup> 4, 2016], <sup>k</sup> e<sup>-</sup> e<sup>-</sup> e<sup>-</sup> i<sup>-</sup> Ki c<sub>i</sub> <sup>1</sup> are e<sup>-</sup> d<sup>-</sup> a<sup>-</sup> Ki c<sub>i</sub> <sup>1</sup> a d<sup>-</sup> k e<sup>-</sup> - i<sup>-</sup> c Ki c<sub>i</sub> <sup>1</sup> re<sup>-</sup> ec<sub>i</sub> e<sup>-</sup>. T<sup>-</sup> e<sup>-</sup> be<sup>-</sup> K r<sup>-</sup> 1 ed e<sup>-</sup> k<sup>--</sup> e<sup>-</sup> fir a<sup>-</sup> r<sup>-</sup> a<sub>i</sub>

The beam K r 1 led e, for the first right and the end of the end

ses, R bid bieari re 6[(2 r)-341.002 (Keedbarc3rd 96 6.002 ()-9.997 ( e eral ed002 ()- K-359 Td[(seed66995 (),

## 3.1 Notation

The stead, e et e bars dat h  $dt \mathbf{K} \in \mathcal{C}_{\mathbf{k}}$  bec<sub>k</sub> (e. ., ca<sup>1</sup>ar, ec<sub>k</sub>r,  $\mathbf{K} \in \mathcal{L}$ ) at  $d = \mathbf{K}$ ercraa de Klec Ida Ka beca Frea let  $\frac{i}{t}$  rescale the let  $\mathbf{K}$  be easy t. Frie a e Kelar, e de  $e^{2}$ e i r b  $\|\cdot\|$ . Feid ced ar in a cared a redefi-fi ed a  $\mathcal{W}[$  ] := arg min<sub> $u \in \mathcal{W}$ </sub> ( -1)<sup>T</sup> ( -1). Fi a<sup>ll</sup>,  $[] := \{1, 2, \ldots, \}.$ 

## 3.2 Learning Model

Wei e-a Krałde-crał Kiełeanti del 1 e-raredi aref.

#### **Problem Formulation**

We-ci defie i becrebadr r ble-i defie GLM real ab a i Ler de relier ber K bec, e a d $\mathcal{X} \subset \mathbb{R}^d$  be e a e. I cal r i d, a cal c c c a a r  $\cdot_t \in \mathcal{X}$  a a d  $\mathcal{X} \subset \mathbb{R}^d$ Least created ectr  $t \in \mathbb{R}^m \operatorname{cl}$  i K beck e. We a e-each beck e  $\frac{i}{t}$  decaded acc rdi  $\frac{i}{t}$  e GLM &  $a_{k} \dot{K}r = 1, 2, ..., ,$ 

$$\Pr(\begin{array}{c}i\\t\end{array}|_{t}) = i\begin{pmatrix}i\\t\end{array}, \tau_{i}) \exp\left(\frac{i\theta_{i}^{\top} + t - i\theta_{i}^{\top} + t}{\tau_{i}}\right)$$

k efe- $\tau_i$  k e-d eff, ara exet, i al r al  $a_i$  i ki  $c_i$  i,  $j_i$  a c i e- ki  $c_i$  i, a d  $\theta_i$  a ec<sub>4</sub> r k i -i c e ki  $c_4$  i. Let  $\mu_i = j_i'$  de set e- called i ki  $c_4$  i.  $\mu_i$  e i  $\mu_i$  creat d e-  $\mu_i$  e-c i e K i L ea  $\mathbb{E}\begin{bmatrix}i\\t\end{bmatrix} = \mu_i(\theta_i^\top t)$ . A re a abe e be K e GLM K i e i e i e

de 1 k & general 1 e-b<sub>4</sub>, i.e.,  $\in \{0, 1\}$  [Z at 1 4, 2016], à d a fie-

$$\Pr(=1|_{-}) = \frac{1}{1 + \exp(-\theta^{\top}_{-})}.$$

Al her ell-1 1 biar de belii, he-GLM herbindel, hera e herklii Kr

$$\mathsf{Pr}(=1|_{\mathsf{P}}) = (\theta^{\top})$$

kere () ke-c laredrobrikicrikke-radardi rald rb i.

#### **Performance Metric**

Accrdi , her ere Khe-GLM, Krat ar  $\in \mathcal{X}$  is a finite of the second relation of the second relation  $\mathbf{K}$  is a finite of the second relation  $\mathbf{K}$  is a finite of the second relation of the second relation  $\mathbf{K}$  is a finite of the second relation of the second relation  $\mathbf{K}$  is a finite of the second relation of the second re et ar b here eckedre and a dad kher ki K Pares rder.

E ed Le Pare, rder, e-cà i defie e Påren ratar. r

Definition 2 (Pareto optimality) 
$$\in \mathcal{X}$$
  $i$   $\ldots$   
 $\lambda_{\mu_{x}} \neq \mu_{x}^{\prime}$ .  $\forall i \neq 1$   
 $\mathcal{X}, \mu_{x} \neq \mu_{x}^{\prime}$ .  $\forall i \neq 1$   
 $\mathcal{X}, \mu_{x} \neq \mu_{x}^{\prime}$ .  $\forall i \neq 1$   
 $\mathcal{O}^{*}$ .

Is clearly as all ar i's e-Pares Kisareic arable. I i le beck ebà dik r ble, he kà dard er ch gearele e eater e Kr à ce re re, defied a le d'Ker-≰a<sup>l</sup>ar.

Definition 3 (Pareto suboptimality gap, PSG)  

$$\mathcal{X}$$
.  
 $\mathcal{X}$ .  

We e a  $a_{1}e^{\frac{1}{2}}e^{-\frac{1}{2}}e^{\frac{1}{2}}e^{\frac{1}{2}}e^{\frac{1}{2}}$  of Kr at  $ce^{-\frac{1}{2}}$   $\frac{1}{2}e^{-\frac{1}{2}}(e^{-\frac{1}{2}}d)$ Parez re-rez [Dr à à d N e, 2013] defi ed a z e-c late-Paret bi alt a Kite-ar Hedbielead er.

Definition 4 (Pareto regret, PR)  

$$\begin{array}{c} & & \\ & & & \\ & & \\ & & & \\ & & \\ & & & \\ &$$

#### 3.3 Algorithm

ter ed a rt, see ed MOGLB-UCB, it ed  $\mathbf{A}$   $\mathbf{A}$  ad à ce, e-c dc se e-Pares Ki drech à daa <sup>111</sup>ge-Paren ratar, ke-Paren brata are-ef. Ma ared b befari, e-airai

at ar  $O_t$  at a r  $a_t$   $a_t$   $b_t$  e Pare,  $K_{1,t}O^*$  at d at a a a r  $O_t$ . To crace Kate, eda at ar t K  $O_t$  r Kr  $a_t$  rad a a  $(S_{4}e^{-3})$ . For a r  $a_te^{-2}$  Pare,  $K_{1,t}$  r  $a_t$  rad  $d_t$  be  $\mathcal{X}$  at  $d_t$   $da_t$  ed a K

٠

I call r I d , affer b cf i i c-re-ard cc<sub>4</sub> r t, e-a c-a c-i a i d c- c d b  $\hat{\theta}_t$  i fread c c fread  $\hat{\theta}_i$  (Sec-4-7). Let  $\mathcal{H}_t := \{(0, 1), (0, 1), \dots, (0, t, t)\}$  be-c-call i c-r r I d . Al as raise rate i c-fread i

$$\hat{\theta}_{t} \quad _{,i} = \underset{\|\theta\| \le D}{\operatorname{arg\,max}} \quad \underset{s=}{\overset{t}{\operatorname{log}}} \operatorname{Pr}\left(\begin{array}{c} i\\s \end{array}\right| \quad _{s}\right)$$
$$= \underset{\|\theta\| \le D}{\operatorname{arg\,min}} \quad \underset{s=}{\overset{t}{\operatorname{log}}} - \begin{array}{c} i\\s \theta^{\top} \quad _{s} + \ldots \\ i(\theta^{\top} \quad _{s}). \end{array}$$

$$t_{t,i}(\theta) := - {i \atop t} \theta^\top t_{t-t} + i (\theta^\top t_{t-t}).$$

Wee a ara, KellieNe i se adc **4**0- $\hat{\theta}_t$  , *i* b

$$\hat{\theta}_{t} \quad _{,i} = \underset{\|\theta\| \le D}{\operatorname{arg\,min}} \frac{\|\theta - \hat{\theta}_{t,i}\|_{Z_{t+1}}}{2} + \theta^{\top} \nabla_{t,i}(\hat{\theta}_{t,i})$$

$$= \frac{Z_{t+1}}{\mathcal{B}_{D}} [\hat{\theta}_{t,i} - \frac{-}{t} \nabla_{t,i}(\hat{\theta}_{t,i})]$$
(1)

k ere-

$$t = t + \kappa$$

 $\mathbf{X}_t^{\top}$ 

Mine in the interview of the second s

**Corollary 1**  $\checkmark$   $r' \geq 0, \gamma = \nu$ 

$$\log \frac{\det \left(\begin{array}{c}t\end{array}\right)}{\det \left(\begin{array}{c}t\end{array}\right)} \leq \log \left(1 + \frac{\kappa}{2\lambda}\right)$$

$$\gamma_t \leq O(\log ).$$

Fiall, e red le Pare, rere, bid Kralrk, ke bl, i i kere 1.

Theorem 2 
$$f = \int \frac{4\pi}{\kappa} \int \frac{4\pi}{\kappa} \int \frac{4\pi}{2\lambda} \int \frac{1-\delta}{\gamma_T}$$
  
()  $\leq 4L = \frac{1-\delta}{\kappa} \log\left(1+\frac{\kappa}{2\lambda}\right)\gamma_T$ 

Remark. The ab effected is a rain of a parent refer bid  $KO(\sqrt{)}$ , the affective a reference bid  $KO(\sqrt{)}$ , the affective reference bid KT refe

Proof of Theorem 2. B F e-re- 1,

$$\theta_i \in \mathcal{C}_{t,i}, \forall \in [], \forall \geq 1$$
 (9)

h  $d = \frac{1}{4}$  r bab $d = \frac{1}{4}$  a $\frac{1}{4}$  ea  $\frac{1}{4} - \delta$ . Freak bec,  $e \in [$ ] a deak r  $1 d \ge 1$ , e-defie-

$$\theta_{t,i} \coloneqq \underset{\theta \in \mathcal{C}_{t,i}}{\operatorname{arg\,max}} \theta^{\top_{t-t}}.$$
(10)

Recall  $\mathbf{k}$   $\mathbf{a}_{\mathbf{k}+t}$  elected  $\mathbf{k}$   $\mathcal{O}_t$ ,  $\mathbf{k}$  is independent of  $\mathbf{k}$  is  $\mathcal{O}_t$ ,  $\mathbf{k}$  is independent of  $\mathbf{k}$  is  $\mathbf{k}$  if  $\mathbf{k}$  is  $\mathcal{O}_t$ ,  $\mathbf{k}$  is  $\mathcal{O}_t$ ,  $\mathbf{k}$  is  $\mathcal{O}_t$ ,  $\mathbf{k}$  is  $\mathbf{k}$  is  $\mathbf{k}$ .

$$\hat{\mu}_{t,x_t}^j \ge \hat{\mu}_{t,x}^j. \tag{11}$$

B defi <u>i</u> i (5) à d (10), et a e-

$$\mu_{t,x_t}^j = \theta_{t,j^{-}t}^\top, \quad \mu_{t,x}^j = \max_{\theta \in \mathcal{C}_{t,j}} \theta^\top \stackrel{()}{=} \vartheta_{j^{+}}^\top.$$
(12)

C bì ì (11) à d(12), e b<sub>4</sub>aì

$$\theta_{t,j^{|} t}^{\top} \ge \theta_{j^{|}}^{\top} . \tag{13}$$

$$\mu_j(\theta_{j+1}^\top) - \mu_j(\theta_{j+1}^\top) \le 0$$

ice- $\mu_j$  i ficall icreat . Frige-lasses cale, eic a e-

$$\begin{split} & \mu_{j}(\theta_{j^{+}}^{\top}) - \mu_{j}(\theta_{j^{+}t}^{\top}) \\ & \leq L(\theta_{j^{+}}^{\top} - \theta_{j^{+}t}^{\top}) \stackrel{(.)}{\leq} L(\theta_{t,j^{+}t}^{\top} - \theta_{j^{+}t}^{\top}) \\ & = L(\theta_{t,j} - \hat{\theta}_{t,j})^{\top_{+}t} + L(\hat{\theta}_{t,j} - \theta_{j})^{\top_{+}t} \\ & \leq L(\|\theta_{t,j} - \hat{\theta}_{t,j}\|_{Z_{t}} + \|\hat{\theta}_{t,j} - \theta_{j}\|_{Z_{t}})\|_{t}\|_{Z_{t}^{-1}} \\ \stackrel{()}{\leq} 2L\sqrt{\gamma_{t}}\|_{t}\|_{Z_{t}^{-1}} \leq 2L\sqrt{\gamma_{T}}\|_{t}\|_{Z_{t}^{-1}} \end{split}$$

Kuj,  $\mathbf{\hat{k}}$  efficit,  $\mathbf{e}$  alt,  $\mathbf{d}$  et  $\mathbf{\hat{k}}$  et  $\mathbf{c}$  it is  $\mathbf{K}_{\mu_j}$ ,  $\mathbf{\hat{k}}$  efficit,  $\mathbf{e}$  alt,  $\mathbf{K}_{\mu_j}$  is  $\mathbf{k}_{\mu_j}$ ,  $\mathbf{k}_{\mu_j}$  efficit,  $\mathbf{k}_{\mu_j}$  is  $\mathbf{k}_{\mu_j}$ .

$$\mu_j(\theta_{j^{\top}}^{\top}) - \mu_j(\theta_{j^{\top}}^{\top}) \le 2L\sqrt{\gamma_T} \|_{t=t} \|_{Z_t^{-1}}.$$

Si ce i e ab e i e al i i d Kr à  $\in \mathcal{X}$ , et a e  $t \leq 2L\sqrt{\gamma_T} \| t \|_{Z_t^{-1}}$ , i e ed a e d a e

$$( ) = \begin{bmatrix} T & & \\ & t \leq 2L\sqrt{\gamma_T} & \\ t = & t = \end{bmatrix} ( t = t = t )$$
 (14)

Webidige-RHS bige-Cale Se ar i e- alige-

B Le- a 11 i Abba - Yad r / & [2011], & a e-

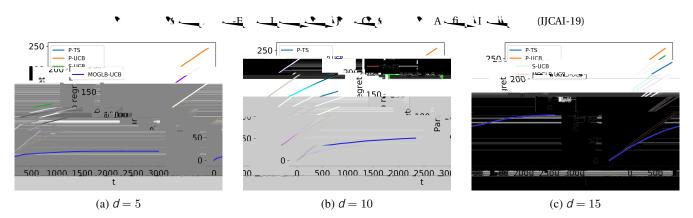
$$\prod_{t=1}^{T} \| \|_{Z_{t}^{-1}} \leq \frac{4}{\kappa} \log \frac{\det(T)}{\det(T)}.$$
 (16)

C bit (14)-(16) at d C r  $^{II}$ ar 1 fi  $^{II}$ e  $^{II}$ e r K  $\Box$ 

## 4 **Experiments**

I  $f_{1}^{2} = e_{f_{1}}^{2} f_{1}^{2} e_{f_{1}}^{2} e_{f$ 

• P-UCB [Dr à à d N e, 2013]: 17 - 4 e-Pare, UCB al r 4 , 4 e c are d'Méré , ar b 4 e-P20163136ade assaig(149369019955 (EQ)9)a36098(2 (W620tme))7890.9



F re-1: Pares re-res Kd Kkerd-s es d

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- [A  $e_{f}$  (2016] Pere A  $e_{f}$  (2 a -Ka (2 a , R) a'd Orl  $e_{f}$  a d Mada'i a Dr à . Pare, Ki , de , ficar i K, , e a , c bà d', Keedbac . I , i , a e 939 947, 2016.
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- [Dr a à d N  $e_7 2014$ ] Madali a M Dr à à d Ali N  $e_7 Scalar a i ba ed are ri a re Kar r dd$ rificari a ri . I ri ri a re france ri a re ri dd-M <math>M = M, a  $e_7 2690 2697, 2014.$
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- [K<sup>1</sup>et bef  $\langle \cdot, 2008$ ] R bef, K<sup>1</sup>et bef, A<sup>1</sup>e à dr S<sup>1</sup> i , à d E<sup>1</sup> U K<sup>1</sup>. M<sup>1</sup> ar ed bà de i err c ace. I  $\langle \cdot, 1 \rangle$  a e 681 690, 2008.
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- $\begin{bmatrix} L & \stackrel{}{\leftarrow} 2019a \end{bmatrix} \overset{\bullet}{S} \stackrel{\bullet}{\leftarrow} L , G \overset{\bullet}{a} \stackrel{\bullet}{\leftarrow} W \overset{\bullet}{a} , Ya H , \overset{\bullet}{a} d \overset{\bullet}{L} \stackrel{\bullet}{\leftarrow} \overset{\bullet}{e} \overset{\bullet}$
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- $\begin{bmatrix} 2 & a & & \\ &$