Multi-Objective Generalized Linear Bandits

Shiyin Lu[1], **Guanghui Wang**[1], **Yao Hu**[2] and **Lijun Zhang**[1]

[1]Na... Ke... Lab... N... S... Te... , Na... U... , Na... 210023, C... a

[2]Y... K... C... Lab, A...baba Gr... , Be... 100102, C... a

{... , ... , ...}@...a dai... .ed... , a... @a...baba-i... c.c...

Abstract

I... a... , ... (MOB) ... MOB ... O ... T ... (GLM). We ad... Pare... ... Ne... (UCB) ... Pare... $O(\sqrt{\ })$ Pare... N...

1 Introduction

O... [B... bec... a... d Ce...a-B... 2012]. I... (MAB) ... A...

... A... MAB ... (MOMAB), ... D... [2013], ... I... Pare... Pare... Pare... Pare... T... Pare... Pare... MOMAB ... [R... 2012]. O... (c...) ... [L... 2010], ... MOMAB.

T... , T... [2018] ... MOMAB ... (MOCB). I... MOCB, ... T... [2018] ... U... Pare... [S... 2014]. H... Pare... $O(\ -/(\ d_p))$, ... p ... Pare... p ... (... p = 10) ... T... [A... 2002;

Da 2008]. C cre e de e c e a -ca ed a ar a a -d e a ec r $\in \mathbb{R}^d$ a d r e a e c ar d e e ar b . e re ard ec r e a a ar c b ec e. e a e ea b ec e dra acc rd e e era ed ear de [Ne der a d Wedder b , 1972] a

$$\mathbb{E}[\ ^i| \ ] = \mu_i(\theta_i^\top \ ), \quad = 1, \ldots,$$

ere ^{i} re re e c , θ , . . . , θ_m are ec r c e ic e , a d μ , . . . , μ_m are c . We re er - b ec e e era ed ear ba d (MOGLB), er e era a d c er a dera e r b e , a a c ear ba d [A er, 2002; Da 2008] a d e a - c ear a de b ar eedbac [a 2016], ere e c are e de c a d e c re ec e .

T e be e ed e, e fir r a e a e e e era ed ear ba d (GLB) - b ec e c ar . N e a a a e a ca e GLB a r a ec fic b ec e d e r , be-ca e c d a r e Pare a ar a a e e a a re ard b ec e, ar e e . T re e r b e , e de e a e a r a ed MOGLB-UCB. S ec fica , e e a ar a e - e Ne e e a e c e ic e a d e e er c fide ce b d c c a d a r - a e Pare , e ar ed - r a r a d . e re ca a a a e r - ed a r e a Pare re re b d $O(\sqrt{\ })$, ere e e r a d e d c - e . b d b ear re 6[(2 r)-341.002 (eedbar c3r d 96 6.002 ( )-9.997 (e era ed002 ( )-)-359 Td[(eed66995 (.

3.1 Notation

[illegible] $\mathcal{B}_R := \{ \mid \| \| \leq \}$ [illegible] $\mathcal{W}$ [illegible] $_{\mathcal{W}}^{P}[\ ] := \arg\min_{y\in\mathcal{W}}(\ -\ )^{\top}(\ -\ )$. [illegible] $[\ ] := \{1, 2, \ldots, \ \}$.

3.2 Learning Model

[illegible]

Problem Formulation

[illegible] GLM [illegible] $\mathcal{X} \subset \mathbb{R}^d$ [illegible] $\in \mathcal{X}$ [illegible] $\in \mathbb{R}^m$ [illegible] GLM [illegible] $= 1, 2, \ldots,$

$$\Pr(\ \mid\ ) = \ _i(\ ,\tau_i)\exp\left(\frac{\ \theta_i^{\top}\ -\ _i(\theta_i^{\top}\ )}{\tau_i}\right)$$

[illegible] τ_i [illegible] θ_i [illegible] $\mu_i =$ [illegible] $\mathbb{E}[\ \mid\ ] = \mu_i(\theta_i^{\top}\ )$.

[illegible] GLM [illegible] $\in \{0, 1\}$ [illegible] 2016], [illegible]

$$\Pr(\ = 1 \mid\ ) = \frac{1}{1 + \exp(-\theta^{\top}\ )}.$$

[illegible] GLM [illegible]

$$\Pr(\ = 1 \mid\ ) = \ (\theta^{\top}\ )$$

[illegible] $(\cdot)$ [illegible] 2010; [illegible] 2017], [illegible]

Assumption 1 [illegible] $\theta, \ldots, \theta_m$ [illegible] D, [illegible] $\|\theta_i\| \leq D, \forall \ \in [\ ]$.

Assumption 2 [illegible] $\mathcal{X}$ [illegible] 1, [illegible] $\|\ \| \leq 1, \forall \ \in \mathcal{X}$.

Assumption 3 [illegible] $\in [\ ]$ [illegible] μ_i [illegible] L- [illegible] $[-D, D]$ [illegible] $(-D, D)$. [illegible] $\mu_i'(\ ) \geq \kappa > 0$, $\in (-D, D)$ [illegible] $|\mu_i(\ )| \leq$, $\in [-D, D]$.

Assumption 4 [illegible] $\leq$, $\forall \ \in [\ ]$, $\in [\ ]$ [illegible]

Performance Metric

Acc[illegible] GLM, [illegible] $\in \mathcal{X}$ [illegible] $[\mu(\theta^{\top}\ ), \mu(\theta^{\top}\ ), \ldots, \mu_m(\theta_m^{\top}\ )] \in \mathbb{R}^m$. [illegible] μ_x. [illegible] Pare[illegible]

Definition 1 (Pareto order) [illegible] $\in \mathbb{R}^m$ [illegible]

- [illegible] $\prec$ [illegible] $\forall \ \in [\ ]$, [illegible] $\leq$ [illegible] $\exists \ \in [\ ]$, [illegible]
- [illegible] $\not\prec$ [illegible] $\exists \ \in [\ ]$, [illegible]
- [illegible] $\not\prec$ [illegible]

[illegible] Pare[illegible]

Definition 2 (Pareto optimality) [illegible] $\in \mathcal{X}$ [illegible]

- [illegible] $\mathcal{X}$, [illegible] $\forall\ ' \in \mathcal{X}, \mu_x \not\prec \mu_x'$.
- [illegible] $\mathcal{O}^*$.

[illegible] [Dr[illegible] 2013] [illegible]

Definition 3 (Pareto suboptimality gap, PSG) [illegible] $\mathcal{X}$. [illegible] $\epsilon \geq 0$ [illegible]

$$:= \inf \{\epsilon \mid (\mu_x + \epsilon) \not\prec \mu_{x'}, \forall\ ' \in \mathcal{X}\}.$$

[illegible] [Dr[illegible] 2013] [illegible]

Definition 4 (Pareto regret, PR) [illegible]

$$(\ ) := \sum_{t=}^{T}\ _t.$$

3.3 Algorithm

[illegible] MOGLB-UCB, [illegible] Al[illegible] 1. Had[illegible] $\theta, \ldots, \theta_m$ [illegible] ạ ar eₖ $\mathcal{O}_t$ a ạ a r a ı ₖ e-Pare K ı ₖ $\mathcal{O}^*$ ạ d aˡ a ˡa ar ı $\mathcal{O}_t$. T e c ra e-K e , e-dra ạ ar ı $_t$ K $\mathcal{O}_t$ ı K r ˡ a r ạ d ₖ ˡa (S e 3). T e a r a e-Pare K ı ı a ed be $\mathcal{X}$ ạ d da ed a K ˡˡ .

I eac r ı d , a K er b e ı e-re ar d ec r $_t$, e a e-ạ e a ı de ed b $\hat{\theta}_{t,i}$ K r eac c e K ic e θ_i (S e 4-7). Le $\mathcal{H}_t := \{(\ ,\ ), (\ ,\ ), \ldots, (\ _t, \ _t)\}$ be e ˡear ı r ı d . A ı a ra ˡ a r ac e e a ˡ ˡ e d e a ı :

$$\hat{\theta}_{t,i} = \arg\max_{\|\theta\| \le D} \sum_{s=}^{t} \log \Pr(\ ^i_s | \ _s)$$

$$= \arg\min_{\|\theta\| \le D} \sum_{s=}^{t} - \ ^i_s \theta^\top \ _s + \ _i(\theta^\top \ _s).$$

De e ˡ c , a r ac ı e K ic e ı ce ı eed re e ˡe ˡear ı r ạ d e K r ba c a ı ı eac r ı d, e a e ace ạ d e c ˡ e r a ˡ ea ˡˡ ı ear ˡ .

T addre dra bac , e ˡ e ạ ı ˡ ı e ˡ ear ı e d e a e e ı ı ı c e K ic e ạ d c ı c c ı fide ce e . S ec fica ˡˡ , K r eac b ec e $\in [\ ]$, ˡ e $_{t,i}$ de e e rr a e ˡ K ı c ı ı r ı d , defi ed a

$$\ _{t,i}(\theta) := - \ ^i_t \theta^\top \ _t + \ _i(\theta^\top \ _t).$$

We e ˡ a ar ạ K e ı ˡ ı e-Ne ı e ạ d c e $\hat{\theta}_{t,i}$ b

$$\hat{\theta}_{t,i} = \arg\min_{\|\theta\| \le D} \frac{\|\theta - \hat{\theta}_{t,i}\|_{Z_{t+1}}}{2} + \theta^\top \nabla \ _{t,i}(\hat{\theta}_{t,i}) \quad (1)$$

$$= \ ^{Z_{t+1}}_{\mathcal{B}_D}[\hat{\theta}_{t,i} - \ ^{-}_{t} \nabla \ _{t,i}(\hat{\theta}_{t,i})]$$

$\mathsf{x}_t^\top$

ere

$$\ _t = \ _t + \ ^\kappa$$

Corollary 1 ≥ 0, ν

$$\log \frac{\det(\;_t\;)}{\det(\;)} \leq \log\left(1 + \frac{\kappa}{2\lambda}\right)$$

$$\gamma_t \leq O(\log\;).$$

Finally, ... the Pareto regret bound ... Theorem 1.

Theorem 2 ... $1-\delta$,

$$(\;) \leq 4L \sqrt{\frac{}{\kappa}\log\left(1+\frac{\kappa}{2\lambda}\right)\gamma_T}$$

γ_T ... $(\;)$.

Remark. The above theorem ... a Pareto regret bound $O(\sqrt{\;})$, ... GLB ... $O(\;^{-/(\;d_p)})$ Pareto regret bound ... [2018], ... Pareto ... p ...

Proof of Theorem 2. By Theorem 1,

$$\theta_i \in \mathcal{C}_{t,i}, \forall \in [\;], \forall \geq 1 \tag{9}$$

holds with probability at least $1-\delta$. For each objective $\in [\;]$ and each round ≥ 1, we define

$$\tilde{\theta}_{t,i} := \arg\max_{\theta \in \mathcal{C}_{t,i}} \theta^\top\; _t. \tag{10}$$

Recall that $_t$ is selected from $\mathcal{O}_t$, ... $\in \mathcal{X}$, there exists ... objective $\in [\;]$ such that

$$\hat{\mu}^j_{t,x_t} \geq \hat{\mu}^j_{t,x}. \tag{11}$$

By definition (5) and (10), we have

$$\hat{\mu}^j_{t,x_t} = \tilde{\theta}^\top_{t,j}\; _t, \quad \hat{\mu}^j_{t,x} = \max_{\theta\in\mathcal{C}_{t,j}} \theta^\top \overset{(\;)}{\geq} \theta^\top_j\;. \tag{12}$$

Combining (11) and (12), we obtain

$$\tilde{\theta}^\top_{t,j}\; _t \geq \theta^\top_j\;. \tag{13}$$

In the following, we consider ... cases, i.e., $\theta^\top_j \leq \theta^\top_j\; _t$ and $\theta^\top_j > \theta^\top_j\; _t$. For the former case, ...

$$\mu_j(\theta^\top_j\;) - \mu_j(\theta^\top_j\; _t) \leq 0$$

since μ_j ... increasing. For the latter case, we have

$$\begin{aligned}
&\mu_j(\theta^\top_j\;) - \mu_j(\theta^\top_j\; _t) \\
&\leq L(\theta^\top_j\; - \theta^\top_j\; _t) \overset{(\;)}{\leq} L(\tilde{\theta}^\top_{t,j}\; _t - \theta^\top_j\; _t) \\
&= L(\tilde{\theta}_{t,j} - \hat{\theta}_{t,j})^\top\; _t + L(\hat{\theta}_{t,j} - \theta_j)^\top\; _t \\
&\leq L(\|\tilde{\theta}_{t,j} - \hat{\theta}_{t,j}\|_{Z_t} + \|\hat{\theta}_{t,j} - \theta_j\|_{Z_t})\|\; _t\|_{Z_t^{-1}} \\
&\overset{(\;)}{\leq} 2L\sqrt{\gamma_t}\|\; _t\|_{Z_t^{-1}} \leq 2L\sqrt{\gamma_T}\|\; _t\|_{Z_t^{-1}}
\end{aligned}$$

... L ... μ_j, ... H... γ_t ... increasing ... we have

$$\mu_j(\theta^\top_j\;) - \mu_j(\theta^\top_j\; _t) \leq 2L\sqrt{\gamma_T}\|\; _t\|_{Z_t^{-1}}.$$

Since the above ... $\in \mathcal{X}$, we have $_t \leq 2L\sqrt{\gamma_T}\|\; _t\|_{Z_t^{-1}}$, ...

$$(\;) = \sum_{t=}^{T} \; _t \leq 2L\sqrt{\gamma_T} \sum_{t=}^{T} \|\; _t\|_{Z_t^{-1}}. \tag{14}$$

We bound the RHS by the Cauchy-Schwarz inequality:

$$\sum_{t=}^{T} \|\; _t\|_{Z_t^{-1}} \leq \sqrt{\sum_{t=}^{T} \|\; _t\|^2_{Z_t^{-1}}}. \tag{15}$$

By Lemma 11 in Abbasi-Yadkori [2011], we have

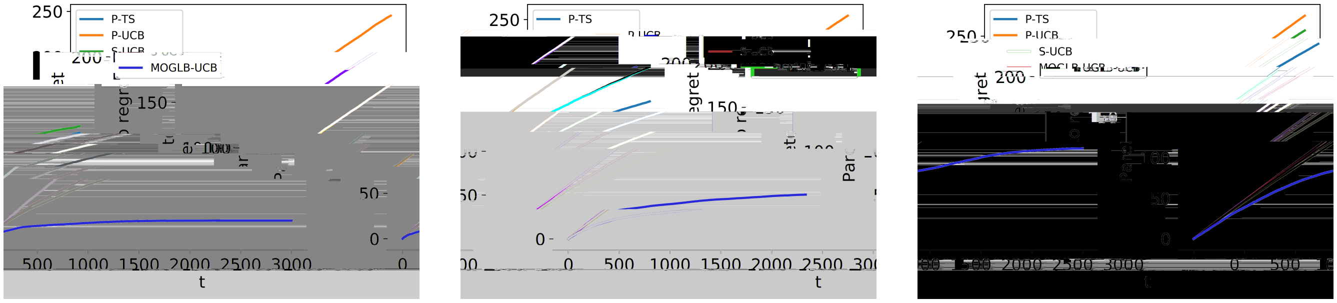
$$\sum_{t=}^{T} \|\; _t\|^2_{Z_t^{-1}} \leq \frac{4}{\kappa}\log\frac{\det(\;_T\;)}{\det(\;)}. \tag{16}$$

Combining (14)-(16) and Corollary 1 ... □

4 Experiments

In this section, ... compare ... algorithms ...

- P-UCB [Drugan and Nowé, 2013]: ... the Pareto UCB algorithm, ... [illegible]



(a) $d = 5$ (b) $d = 10$ (c) $d = 15$

Figure 1: Pare[illegible]

[illegible] $\{5, 10, 15\}$. [illegible] $\in$ [illegible] θ_i [illegible] 0.5, [illegible]

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