



Non-stationary Dueling Bandits for Online Learning to Rank

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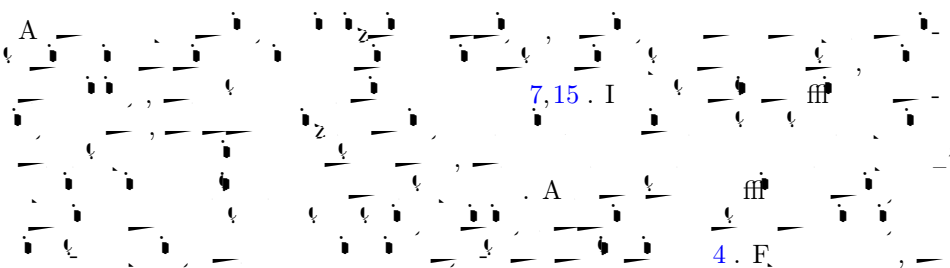
² A baba G , Ha 311121, C a

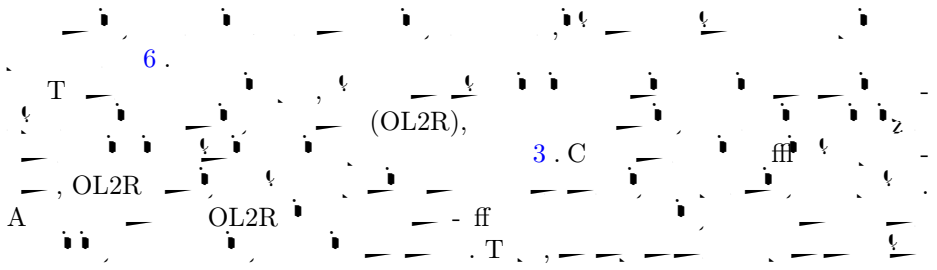
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Ab ac. W s d y a a r (OL2R), a a a -
d a r d b a s d s a d b a o
s s A a a a d a a a c OL2R a a a s
a a a d d b a d s b , a c a q s d s
a a r , a r d a s c c a a q a -
W d b a d s a d s a c a OL2R a b
s s d d a , s r s c s s a c
c d a r s s s d b s a -
a y H , s s a d a d OL2R
a c a s s c y c a c a s a d s d s
a a a r . T a d d s s y s b , s s q s s a a y
d b a d s r c d a r s s d d b y
a - a a c . W d a f f i c a d a d a d
e - s a a y d b a d s s s c a a a s
T a d a d s d b a d s a d
d (DBGD) a s d f s s a a a d
y a a a d y a c a y q b s DBGD a -
s a c c d a a a c . W s a d y a d
s s d c a a s a y s DBGD - y a -
s

Ke d : O a a a r . D b a d s
N s a a y s

1 Introduction





2 Problem Setup

Let $\mathcal{W} \subseteq \mathbb{R}^L$ be a set of vectors, $T = \{1, \dots, T\}$ a set of indices, and $f: \mathcal{W} \times \mathcal{W} \rightarrow \mathbb{R}$ a function. For $t \in T$, let $v: \mathcal{W} \rightarrow \mathbb{R}$ be a function. We assume f and v are symmetric and bounded. Specifically, we assume $f(\mathbf{w}, \mathbf{w}') = \sigma(v(\mathbf{w}) - v(\mathbf{w}'))$ for some sigmoid function σ . We also assume $\max_{\mathbf{w} \in \mathcal{W}} \|\mathbf{w}\|_2 \leq R$.

$$\Pr(\mathbf{w} \succ \mathbf{w}' | t) = f(\mathbf{w}, \mathbf{w}') = \sigma(v(\mathbf{w}) - v(\mathbf{w}')) \quad (1)$$

$$\max_{\mathbf{w} \in \mathcal{W}} \|\mathbf{w}\|_2 \leq R. \quad (2)$$

$$\sigma(x) = 1 - \sigma(-x). \quad (3)$$

$$\sigma(-\infty) = 0, \quad \sigma(0) = 1/2, \quad \sigma(\infty) = 1.$$

Let $\mathbf{w}^* = \arg \max_{\mathbf{w} \in \mathcal{W}} v(\mathbf{w})$ be the vector that maximizes v over \mathcal{W} . We assume $\mathbf{w}^* \in \mathcal{W}$.

$$DR(T) = \sum_{t=1}^T (f(\mathbf{w}^*, \mathbf{w}_t) + f(\mathbf{w}_t, \mathbf{w}^*) - 2f(\mathbf{w}^*, \mathbf{w}^*)).$$

² I OL2R, a id y sd r c s r id c $\sigma(x) = 1/(1+\exp(-x))$,
 $\sigma(x) = \frac{1}{1 + \exp(-x)}$

3 Method

I DBGD- (DBGD)

3.1 a a a

A A, 1, DBGD δ γ I
 t , DBGD \mathbf{u} T

$$\mathbf{w}' = \Pi_{\mathcal{W}}[\mathbf{w} + \delta \mathbf{u}] \tag{4}$$

\mathbf{w} \mathbf{w}' \mathcal{W} . N $\Pi_{\mathcal{W}}[\cdot]$ 5,
 S, DBGD \mathbf{w}' F,
 O \mathcal{W} \mathbf{z} DBGD $\gamma: \mathbf{w}_{+1} = \Pi_{\mathcal{W}}[\mathbf{w} + \gamma \mathbf{u}]$.

1. Let C be the path length of the optimal rankers over T rounds, defined as

$$C = \sum_{=2} \|\mathbf{w}^* - \mathbf{w}^*_{-1}\|_2. \tag{5}$$

By setting $\delta = \sqrt{\frac{2}{(11+2)}} \sqrt{\lambda}$ and $\gamma = \sqrt{\frac{5^{-2+2}}{T}}$, the dynamic regret of DBGD satisfies

$$[\text{DR}(T)] \leq \sqrt{2(11 + 2\lambda)\lambda dL} \left(1 + \sqrt{5R^2 + 2RC}\right) T^{\frac{3}{4}}.$$

3.2 a a a

\mathcal{W} DBGD $C = o(\sqrt{T})$,
 C γ

1. DBGD

-
- 1: $I \leftarrow \mathbf{a} \text{ a r } \mathbf{w}_1 \in \mathcal{W} \text{ a b a } \mathbf{y}$
 - 2: $f \leftarrow t = 1, 2, \dots, T \text{ d}$
 - 3: $D \leftarrow \mathbf{a} \text{ c } \mathbf{u}_t \text{ y a a d } \mathbf{e}$
 - 4: $C \leftarrow \mathbf{a} \text{ a } \mathbf{y} \text{ a r } \mathbf{w}'_t = II_{\mathcal{W}}[\mathbf{w}_t + \delta \mathbf{u}_t]$
 - 5: $G \leftarrow \mathbf{a} \text{ w}_t \text{ a d } \mathbf{w}'_t \text{ b y } \mathbf{b a b a s c} \text{ a}$
 - 6: $\text{if } \mathbf{w}'_t \succ \mathbf{w}_t \text{ e}$
 - 7: $S \leftarrow \mathbf{w}_{t+1} = II_{\mathcal{W}}[\mathbf{w}_t + \gamma \mathbf{u}_t]$
 - 8: e e
 - 9: $S \leftarrow \mathbf{w}_{t+1} = \mathbf{w}_t$
 - 10: e d f
 - 11: e d f
-



Meta Algorithm A A 2, ff $z \gamma \cdot A^t$

$$N = \left\lceil \log_2 \sqrt{1 + 4T/5} \right\rceil + 1 \tag{6}$$

$$\gamma = 2^{-1} R \sqrt{5/T}, \quad i = 1, \dots, N. \tag{7}$$

E^k $i \in [N]$ π , $i \cdot F$

$$\pi_1 = \frac{N+1}{i(i+1)N}, \quad i = 1, \dots, N. \tag{8}$$

I t , \mathbf{w} $i \in [N]$ $\pi, i \in [N] \leftarrow \mathbf{w} =$

$\sum_{i=1}^N \pi_i \mathbf{w}_i \cdot \mathbf{T}$, $\mathbf{w}' = \Pi_{\mathcal{W}}[\mathbf{w} + \delta \mathbf{u}]$, $\{w'_i > w_i\} \cdot N$,

$$\pi_{i+1} = \frac{\pi_i \exp(-\alpha \ell(\mathbf{w}_i))}{\sum_{j=1}^N \pi_j \exp(-\alpha \ell(\mathbf{w}_j))}, \quad i = 1, \dots, N \quad (9)$$

$\ell(\mathbf{w})$

$$\ell(\mathbf{w}) = -\frac{d}{\delta} \langle \{w'_i > w_i\} \mathbf{u}, \mathbf{w} - \mathbf{w}' \rangle$$

$\{w'_i > w_i\} \mathbf{u}$

Expert Algorithm. A DBGD. I t , A , 3 , $i \in [N]$, $\{w'_i > w_i\} \mathbf{u}$

$$\mathbf{w}_{i+1} = \Pi_{\mathcal{W}}[\mathbf{w}_i + \gamma \{w'_i > w_i\} \mathbf{u}]$$

2. DM²L: M —A,

- Re e: b $\gamma_1, \dots, \gamma_N, a, \alpha$
- 1: I A $\gamma_i, i \in [N]$
 - 2: I $\pi_i, i \in [N]$ b (8)
 - 3: f $t = 1, 2, \dots, T$ d
 - 4: R c a v $w_t^i, i \in [N]$
 - 5: A a a v $w_t = \sum_{i=1}^N \pi_i w_t^i$
 - 6: D a a c u_t, y, a, d
 - 7: C a a a v $w_t' = \Pi_W[w_t + \delta u_t]$
 - 8: C a w_t a d w_t', b, a, c
 - 9: U da $\pi_i, i \in [N]$ b (9)
 - 10: S d $\{w_t' > w_t\}$ a d $u_t, i \in [N]$
 - 11: e d f

3. DM²L: E A,

- Re e: γ_i
- 1: I a a a v $w_i \in \mathcal{W}$ a b a y
 - 2: f $t = 1, 2, \dots, T$ d
 - 3: S d a v w_t^i, A 2
 - 4: R c $\{w_t' > w_t\}$ a d u_t, A 2
 - 5: U da a v $w+11$

Ac ed e e . T s r v a s a a y s e d by NSFC (61976112) a d
 Ja sSF (BK20200064). W a r a e y e s a r s e a q s c
 s s s

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