Th Thirty- i th I on r n on rti i i l Int llig n (I-21)

Stochastic Graphical Bandits with Adversarial Corruptions

Shiyin Lu, Guanghui Wang, Lijun Zhang*

210023,

Abstract

$O(\alpha \ln K \ln T + \alpha C)$, α , K , T , C

Introduction

2016), 2015), 2017). T

(2012, 2020). (2011). (2017). (2006) (2011), (2012, 2014, 2016, 2017, 2018, 201 2018, 2020), 2011, 2013, 2014, 2015, 2015, 2016, 2018, 2018, 201 2020),

$\sqrt{T}$

(2014)

2018 201

201) (2020),

2018

201),

( )

(200).

201),

$(\alpha \ln \ln T + \alpha)$ α

$(\ln \ln T +)$

α

500

$\alpha = 1$,

(2020).

Related Work

Stochastic Graphical Bandits

(2012),

(2012) $(\ln T +$)

(201)

$(\ln T)$

(2014)

$\gamma \ln T +$ γ

(2016)

(2016)

α ln $\ln(T)$ α

T

(, 1 33).

(2017) ( ) 1 2 0.003 12 -68.8 6 ,8288.

(201) (ln ln T +) AA

(201) $\Omega($ ln $T+$)

(201),

(ln T +)

(A . 2020), (201), (2020), (201 201), (. 201).

fi (A 201 201 . 2020).

(2012. 2014. A 2016. 2017. 2018. 201 201 201).

(. 2018. . 2018. 201 2020. . 2020).

Problem Setup

[ ] = {1 } = (V). V = [ ] $\subseteq V \times V$. $\in$ [ ], ( ).

( ) = { } ∪ ′ ∈ [ ] | (′) ∈

T ,

(1) A. ( ) ∈ [0 1]

(2) ( ) $_{a\in[K]}$ ( ) ∈ [0 1] ∈ [ ].

(3) ∈ [ ] ( ) ( ) ( ) ∈

(201), ( ). ∈ [ ] * ∈ arg max$_{a\in[K]}$ ( ). fi

$$\mathrm{R}(T) = \sum_{=1} (\ ^*) - \sum_{=1} (\ ) = \sum_{=1} \Delta(\ ) \quad (1)$$

$\Delta(\ ) = (\ ^*) - (\ )$. $\in$ [ ].

$$= \sum_{=1} \max_{a\in[K]} |\tilde{\ }(\ ) - (\ )| \quad (2)$$

fi . (. 2001).

Definition 1 (Independent Set) *An independent set $\mathcal{I}$ in a graph $= (V\ \ )$ is a subset of V such that no two vertices in $\mathcal{I}$ are adjacent.*

Definition 2 (Independence Number) *The independence number α of a graph is the cardinality of the largest independent set in .*

Algorithm

(AA) A A (201). fi AA A A AA $\mathcal{S} \subseteq$ [ ] T

( ) $\mathcal{S}$ $\mathcal{S}$ ($|\mathcal{S}|$).

( ) $\mathcal{S}$ fi

(ln T) $\mathcal{S}$ $|\mathcal{S}| \leq$ 1 (ln T), A AA

Algorithm 1

Input: fi $\delta \in (0, 1)$, T

1. $\leftarrow 1$, $\tau_1 \leftarrow 1$, $\leftarrow 1$, $\tilde{\Delta}_0(\ ) \leftarrow 1 \ \forall \ \in [\ ]$
2. $\lambda \leftarrow 273 \ln(3 \ \delta^{-1} \log_2 T)$
3. **while** $\leq T$ **do**
4. 2 $\tilde{\Delta}_{m-1}(\ )_{a\in[K]}$ $\mathcal{I}_m$
5. $\tau_{m+1} \leftarrow \min\ T+1\ \ \tau_m + \ \lambda \sum_{a\in\mathcal{I}_m} \tilde{\Delta}_{m-1}(\ )^{-2}$
6. m (3)
7. **while** τ_{m+1} **do**
8. $\sim\ m$ $\{\tilde{\ }(\ )\}_{a\in\phi(I_t)}$
10. $\leftarrow \ +1$
11. **end while**
12. **for** $= 1$ **do**
13. $\bar{\ }_m(\ )$ (4)
14. **end for**
15. $\hat{\ }^*_m \leftarrow \max_{a\in[K]} \bar{\ }_m(\ ) - \tilde{\Delta}_{m-1}(\ )\ 10$
16. **for** $= 1$ **do**
17. $\tilde{\Delta}_m(\ ) \leftarrow \max\ 2^{-m}\ \hat{\ }^*_m - \bar{\ }_m(\ )$
18. **end for**
1 $\leftarrow \ +1$
20. **end while**

(201) $\tilde{\Delta}(\ )$ 0 [ ] $\tilde{\Delta}(\ )^{-2}$ (201) (ln ln T + (α ln ln T + α), α $\mathcal{I}$ [ ], ( ) (α).

Algorithm 2

Input: $\tilde{\Delta}(\ )_{a\in[K]}$

1. $\mathcal{A} \leftarrow [\ ]$, $\mathcal{I} \leftarrow \varnothing$
2. **repeat**
3. $\in \arg\min_{a'\in\mathcal{A}} \tilde{\Delta}(\ ')$
4. $\mathcal{I} \leftarrow \mathcal{I} \cup \{\ \}$
5. $\mathcal{A} \leftarrow \mathcal{A} - (\ )$
6. **until** $\mathcal{A} = \varnothing$
7. **return** $\mathcal{I}$

(2016), fi $\mathcal{A} = [\ ]$ $\mathcal{I} = \varnothing$, $\mathcal{I}$ $\mathcal{A} = \varnothing$ $\mathcal{A}$ $\mathcal{I}$, $\mathcal{A}$. 2 $\in [\ ]$ 2 $\in \mathcal{I}$ ( ) $\in$ $\tilde{\Delta}(\ ) \leq \tilde{\Delta}(\ )$. ( ) ( ) $\mathcal{I}$, $\in [\ ]$, 1. $\tilde{\Delta}_m(\ )$ $= 1\ 2$ fi $\mathcal{I}_m$ 2 $\tilde{\Delta}_{m-1}(\ )_{a\in[K]}$ $\mathcal{I}_m$ m $\in \mathcal{I}_m$, $\lambda\ \tilde{\Delta}_{m-1}(\ )^{-2}$ $\lambda \sum_{a\in\mathcal{I}_m} \tilde{\Delta}_{m-1}(\ )^{-2}$ m

$$
{}_m(\ ) = \begin{cases} \dfrac{\left(\tilde{\Delta}_{m-1}(a)\right)^{-2}}{\sum_{a'\in\mathcal{I}_m}\left(\tilde{\Delta}_{m-1}(a')\right)^{-2}} & \in \mathcal{I}_m \\ 0 & \notin \mathcal{I}_m \end{cases} \tag{3}
$$

$(\ ) = \mathbb{1}\{\ \in (\ )\}$ $\in [\ ]$, ${}_m(\ )$

$$
\bar{\ }_m(\ ) = \frac{\sum_{=\tau_m}^{\tau_{m+1}-1} \tilde{\ }(\ )\ (\ )}{\sum_{=\tau_m}^{\tau_{m+1}-1} (\ )} \tag{4}
$$

m fi 0 $0 = 0$. fi $\hat{\ }^*_m$

(*) $\tilde{\Delta}_m($)

∈[ ] $_m($) fi $\hat{}^*_m$.

A A ,

fi A A

- A , A A ∈ [ ].

$$-'_m(\ ) = \frac{\sum_{=\tau_m}^{\tau_{m+1}-1} \tilde{}(\ )\ (\ )}{\mathbb{E}\ \sum_{=\tau_m}^{\tau_{m+1}-1}\ (\ )} \qquad (5)$$

A A $^-_m($) fi (4)

fi $\hat{}^*_m$

$\tilde{\Delta}_m($)$_{a\in[K]}$. (5)

$^{-'}_m($) 1

$\tilde{\Delta}_m($)

1. A A (

4 , , 201)

$\tilde{\Delta}_m($) ≤ 1

(5) (4),

$^-_m($) m $^a_{\in K}$

Proposition 1 *The following two facts hold.*

(i) $\leq \log_4 T$ *and* $\tilde{}_m \geq \lambda 2^{2(m-1)}$ $\forall \in [\quad - 1]$

(ii) With probability at least $1-\delta$*, for all arms* $\in [\quad]$ *and all epochs* $\in [\quad]$*, we have* $\frac{\tilde{n}_m(a)}{n_m(a)} \leq \frac{12\kappa}{11}$ *and*

$$\tilde{\Delta}_{m-1}(\ ) \geq \frac{8\Delta(\ )}{9} - \frac{12}{5} \cdot 2^{-m} - 3 \ {}_{m-1}$$

where we define $= \frac{501}{500}$ *and*

$$_m = \sum_{=1}^{m} \frac{2\ 2}{5^{m-}\ \tilde{}} \qquad (8)$$

Proof of Theorem 1

1. fi

$$\mathrm{R}(T) = \sum_{=1}^{T} \Delta(\ ) = \sum_{m=1}^{M} \sum_{=\tau_m}^{\tau_{m+1}-1} \Delta(\ ) \qquad (\ )$$

1, $\in [\quad]$, $\mathcal{I}_m$ $\mathcal{I}_m$ $\in \mathcal{I}_m$, $\in [\ ]$,

$$\sum_{=\tau_m}^{\tau_{m+1}-1} \Delta(\ ) = \sum_{a \in \mathcal{I}_m} \Delta(\ )\tilde{}_m(\ ) = \sum_{a \in \mathcal{I}_m - \mathcal{A}^*} \Delta(\ )\tilde{}_m(\ )$$
$$\leq \frac{12}{11} \sum_{a \in \mathcal{I}_m - \mathcal{A}^*} \Delta(\ )\ {}_m(\ ) \qquad (10)$$

1 $\mathcal{A}^*$ fi 1. $\in [\ ]$ $\in \mathcal{I}_m - \mathcal{A}^*$.

(1) $0 \quad \Delta(\ ) \leq 4\ 2^m$. , ${}_m(\ )$

$$_m(\ ) = \lambda \sum_{a' \in \phi(a) \cap \mathcal{I}_m} \tilde{\Delta}_{m-1}(\ ')^{-2}$$
$$= \lambda\ \tilde{\Delta}_{m-1}(\ )^{-2} \leq \lambda 2^{2(m-1)} \leq \frac{4\lambda}{\Delta(\ )^2}$$

$\Delta(\ )\ {}_m(\ ) \leq \frac{4\lambda}{\Delta(a)}$

(2) $\Delta(\ ) \quad 4\ 2^m$ ${}_{m-1} \leq \Delta(\ )\ 36.$, 1,

$$\tilde{\Delta}_{m-1}(\ ) \geq \frac{8\Delta(\ )}{9} - \frac{12}{5} \cdot 2^{-m} - 3\ {}_{m-1}$$
$$\geq \left(\frac{8}{9} - \frac{3}{5} - \frac{1}{12}\right) \Delta(\ ) \geq \frac{\Delta(\ )}{5}$$

[1]

$$\Delta(\ )\ {}_m(\ ) = \Delta(\ )\lambda\ \tilde{\Delta}_{m-1}(\ )^{-2} \leq \frac{25\lambda}{\Delta(\ )}$$

(3) $\Delta(\ ) > 4\ 2^m$ ${}_{m-1} > \Delta(\ )\ 36.$,

$$\Delta(\ )\ {}_m(\ ) \leq 36\ {}_{m-1}\ {}_m(\ ) = 36\ {}_{m-1}\lambda\ \tilde{\Delta}_{m-1}(\ )^{-2}$$
$$\leq 36\lambda\ {}_{m-1} 2^{2(m-1)} \leq 9\lambda\ {}_{m-1} 2^{2m}$$

$\in \mathcal{I}_m - \mathcal{A}^*$

$$\Delta(\ )\ {}_m(\ ) \leq \frac{25\lambda}{\Delta(\ )} + 9\lambda\ {}_{m-1} 2^{2m}$$

(10)

$$\sum_{=\tau_m}^{\tau_{m+1}-1} \Delta(\ ) \leq \frac{12}{11} \sum_{a \in \mathcal{I}_m - \mathcal{A}^*} \left(\frac{25\lambda}{\Delta(\ )} + 9\lambda\ {}_{m-1} 2^{2m}\right)$$
$$\leq \frac{300\ \lambda}{11} \sum_{a \in \mathcal{I}^* - \mathcal{A}^*} \frac{1}{\Delta(\ )} + \frac{108\alpha\ \lambda}{11}\ {}_{m-1} 2^{2m}$$

$\mathcal{I}^*$ fi (6). ( ),

$$\mathrm{R}(T) \leq \frac{300\ \lambda}{11} \sum_{a \in \mathcal{I}^* - \mathcal{A}^*} \frac{}{\Delta(\ )} + \frac{108\alpha\ \lambda}{11} \sum_{m=1}^{M}\ {}_{m-1} 2^{2m}$$
$$\leq \frac{300\ \lambda}{22} \sum_{a \in \mathcal{I}^* - \mathcal{A}^*} \frac{\log_2 T}{\Delta(\ )} + \frac{108\alpha\ \lambda}{11} \sum_{m=1}^{M}\ {}_{m-1} 2^{2m}$$

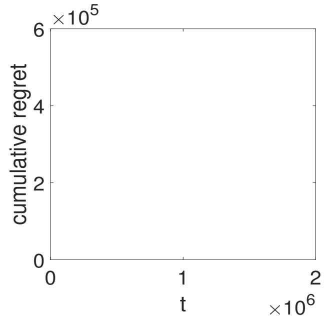
1. $\sum_{m=1}^{M}\ {}_{m-1} 2^{2m}$. 1 fi ${}_m$ (8),

$$\sum_{m=1}^{M}\ {}_{m-1} 2^{2m} = \sum_{m=1}^{M} 2^{2m} \sum_{=1}^{m-1} \frac{2\ 2}{5^{m-1-}\ \tilde{}}$$
$$\leq \sum_{m=1}^{M} 2^{2m} \sum_{=1}^{m-1} \frac{2\ 2}{5^{m-1-}\ \lambda 2^{2(\ -1)}}$$
$$= \frac{35\ 2}{\lambda} \sum_{m=1}^{M} \sum_{=1}^{m-1} (4\ 5)^{m-1-}$$
$$= \frac{35\ 2}{\lambda} \sum_{=1}^{M-1} \sum_{m=\ +1}^{M} (4\ 5)^{m-1-}$$
$$\leq \frac{35\ 2}{\lambda} \sum_{=1}^{M-1} \sum_{h=0}^{+\infty} (4\ 5)^h \leq \frac{176}{\lambda}$$

$\lambda = 273 \ln(3 \quad \delta^{-1} \log_2 T)$ $= 501 \quad 500$,

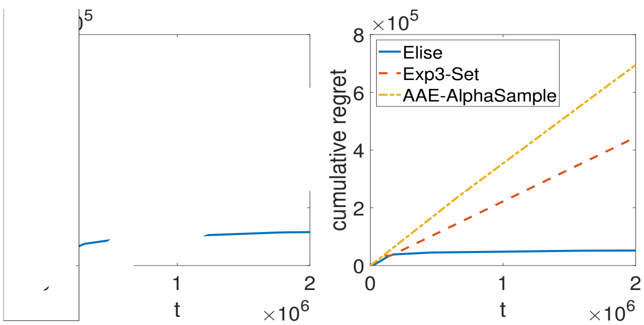
$$\mathrm{R}(T) \leq 1732\alpha \quad + 3731 \ln(3 \quad \delta^{-1} \log_2 T) \sum_{a \in \mathcal{I}^* - \mathcal{A}^*} \frac{\log_2 T}{\Delta(\ )}$$

fi 1. □



( ) $K = 5$ ( ) $K = 10$

1.



( ) $p = 0.2$ ( ) $p = 0.5$

2. 3-

Experiments

Multi-Armed Bandits

fi

$\alpha =$

fi

$\in \{1\ 2\}$,

( ) $= 0\ 8$. $\in$

$\{3$ $\}$,

$[0\ \ 0\ 6]$. $\in [\ ]$,

$\{\ (\ )\}_{\in [T]}$

$[0\ 1]$, ( ), $0\ 01$.

$= 2000000$

$= 1000 \ln(T)$.

(201),

fi $\in [T]$,

10

1.

(201). fi

(ln ln T +)

Graphical Bandits

$\alpha \neq$

$= 10$

(1 60). fi-

( ) $\in [\ ] \times [\ ]$ $\neq$

fi

(2016)

3- (. 2013).

10

2.

fi

$(\alpha \ln \ \ln T + \alpha\ )$

Conclusion and Future Work

fi

fi

(2013. . 201).

( )

Acknowledgments

[illegible] (2017 [illegible] 1002201), [illegible] (20200064), [illegible]

References

[illegible] 2015. [illegible] *Proceedings of the 28th Conference on Learning Theory*, [illegible] 40.

[illegible] 2017. [illegible] *SIAM Journal on Computing* 46(6): 1785–1826.

[illegible] 2013. [illegible] *Advances in Neural Information Processing Systems 26*, 1610–1618.

[illegible] 201 . [illegible] *Journal of Machine Learning Research* 20(1): 1– 3 .

[illegible] 2020. [illegible] *arXiv preprint arXiv:2002.10286* .

[illegible] 201 . [illegible] *Advances in Neural Information Processing Systems 32*, 103 7–10407.

[illegible] 2016. [illegible] *Proceedings of the 29th Conference on Learning Theory*, 116–120.

[illegible] 1 24. [illegible] *Ann. Sci. Inst. Sav. Ukraine, Sect. Math* 1(4): 38–4 .

[illegible] 2020. [illegible] *Proceedings of the 23rd International Conference on Artificial Intelligence and Statistics*, 1071–1081.

[illegible] 2012. [illegible] *Foundations and Trends in Machine Learning* 5(1): 1–122.

[illegible] 2013. [illegible] *IEEE Transactions on Information Theory* 5 (11): 7711–7717.

[illegible] 2012. [illegible] *Proceedings of the 25th Conference on Learning Theory*, 42 1.

[illegible] 2014. [illegible] *Proceedings of the 2014 ACM International Conference on Measurement and Modeling of Computer Systems*, 28 –300.

[illegible] 2012. [illegible] *Proceedings of the 28th Conference on Uncertainty in Artificial Intelligence*, 142–151.

[illegible] 2006. *Prediction, Learning, and Games.* [illegible]

[illegible] 201 . [illegible] *arXiv preprint arXiv:1910.04183* .

[illegible] 2016. [illegible] *Proceedings of the 33rd International Conference on Machine Learning*, 811–81 .

[illegible] 200 . *Concentration of measure for the analysis of randomized algorithms.* [illegible]

[illegible] 1 60. [illegible] *Publ. Math. Inst. Hung. Acad. Sci* 5(1): 17–60.

[illegible] 2006. [illegible] *The Journal of Machine Learning Research* 7( ): 107 –1105.

[illegible] 2018. [illegible] *2018 IEEE International Symposium on Information Theory (ISIT)*, 31 –35.

[illegible] 2020. [illegible] *arXiv preprint arXiv:2002.03839* .

[illegible] 2020. [illegible] *Proceedings of the 34th AAAI Conference on Artificial Intelligence.*

[illegible] 201 . [illegible] *Proceedings of the 32nd Conference on Learning Theory*, 1562–1578.

[illegible] 201 . [illegible] *Proceedings of the 35th Conference on Uncertainty in Artificial Intelligence.*

[illegible] 2018. [illegible] *Advances in Neural Information Processing Systems 31*, 3640–364 .

[illegible] 201 . [illegible] *Machine Learning* 108(4): 687–715.

[illegible] 2016. [illegible] *Proceedings of the 19th International Conference on Artificial Intelligence and Statistics*, 1186–11 4.

[illegible] 2014. [illegible]

Advances in Neural Information Processing Systems 27, 613 621.

2020. *Bandit Algorithms.*

2020. *Proceedings of the 33rd Conference on Learning Theory*, 2516 2564.

201 . *arXiv preprint arXiv:1909.02109* .

2018. *Proceedings of the 32nd AAAI Conference on Artificial Intelligence*, 3643 3650.

201 . *Proceedings of the 36th International Conference on Machine Learning*, 4042 4050.

2018. *Proceedings of the 34th Conference on Uncertainty in Artificial Intelligence*, 13 22.

2020. *Proceedings of the 45th IEEE International Conference on Acoustics, Speech and Signal Processing*, 3112 3116.

201 . *Proceedings of the 36th International Conference on Machine Learning*, 4154 4163.

2018. *Proceedings of the 50th Annual ACM SIGACT Symposium on Theory of Computing*, 114 122.

201 . *arXiv preprint arXiv:1911.08689* .

2018. *Proceedings of the 31st Conference on Learning Theory*, 7 86.

2020. *Proceedings of the 31st International Conference on Algorithmic Learning Theory*, 5 2 614.

2018. *Proceedings of the 8th International Conference on Decision and Game Theory for Security*, 186 204.

2016. *IEEE Wireless Communications* 23(3), 64 73.

2011. *Advances in Neural Information Processing Systems 24*, 684 6 2.

201 . *Journal of Machine Learning Research* 20(83), 1 28.

2015. *Advances in Neural Information Processing Systems 28*, 3168 3176.

201 . *Proceedings of the 22nd International Conference on Artificial Intelligence and Statistics*, 2435 2444.

2017. *Marketing Science* 36(4), 500 522.

2017. 3++ *Proceedings of the 30th Conference on Learning Theory*, 1743 175 .

2014. *Proceedings of the 31st International Conference on Machine Learning*, 1287 12 5.

1 33. *Biometrika* 25(3/4), 285 2 4.

2017. *Proceedings of the 31st AAAI Conference on Artificial Intelligence*.

2015. *Statistical Science* 30(2), 1 215.

2018. *Proceedings of the 31st Conference On Learning Theory*, 1263 12 1.

2001. *Introduction to graph theory*, 2.

200 . *Marketing Science* 28(2), 2 3 308.

201 . *Proceedings of the 36th International Conference on Machine Learning*, 7683 76 2.

201 . *Proceedings of the 22nd International Conference on Artificial Intelligence and Statistics*, 467 475.